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VALIDATION OF THE DESIGN METHODOLOGY FOR SUBMARINES;  
A COMPARISON OF ANALYSES AND EXPERIMENTS  
PART 1

15 June 1993  
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## SUMMARY

During the past five years considerable progress has been made with regard to insight in the structural design and analysis of submarine pressure hulls, particularly as far as stability and plasticity are concerned (ref. [2], [3], [5] - [12]). In 1991 a design methodology has been developed (ref. [1]). This design methodology makes the knowledge and experience available to the designer in the form of design directives for the pressure hull.

For validation of the methodology, it is important to compare results from the methodology with experimental results. This report compares the results of analyses (for all collapse modes) as prescribed by the methodology with experimental results of three aluminium cylinders as tested in Canada (ref. [13] - [15]).

Some important conclusions from chapter 5 are:

- Using the additional remarks on the design methodology as given in section 3.1, two important aims of the design methodology have been met, namely:
- real (corrected) pressure values obtained with analytical methods for the different failure modes are the smallest ones. So continuing from stage 2 to stage 3 - 5 of the design methodology, no modifications are necessary unless the weight has to be reduced. Using more sophisticated analysis methods results in an increasing accuracy.
- the resulting errors (difference between corrected collapse pressure and experimental collapse pressure) are small, and an underestimation of the experimental collapse pressure is obtained. These errors are, for CYL 1, CYL 2 and CYL 5 respectively (in percents, positive is overestimation of collapse pressure):

3.6 /	7.5 /	3.2	calculated collapse pressures for measured imperfections.
-10.2 /	-6.9 /	-10.6	real (corrected) collapse pressures for measured imperfections.
5.0 /	10.8 /	9.4	calculated collapse pressures for smallest amplitude (Fourier analysis) for n=3 imperfections.

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- 18.1 / -13.6 / -14.6 real (corrected) collapse pressures for smallest amplitude (Fourier analysis) for n=3 imperfections.
- 9.0 / -2.1 / -2.9 calculated collapse pressures for largest amplitude (on circumference) for n=3 imperfections.
- 29.0 / -23.6 / -24.3 real (corrected) collapse pressures for largest amplitude (on circumference) for n=3 imperfections.

It is obvious that calculated collapse pressures will result in an overestimation, unless the maximum amplitude as found on the circumference is supposed to be present in the worst buckling mode. Of course, this will always result in an underestimation. Using the Fourier term for n=3 as amplitude for the n=3 imperfection, and correcting these collapse pressures for remaining uncertainties and inaccuracies, results in an underestimation (design margin, see ref. [1]) of 13.6 to 18.1 percent. Using the measured imperfections (more sophisticated analyses, reduction of uncertainties) results in a design margin of only 6.9 to 10.6 percent.

- Although the design margins above are sufficiently small, which was one of the aims of the design methodology, the amount of reduction of the design margin is strongly dependent of the knowledge of type and amplitude of the imperfection. As the results above show, a factor 1.69 to 2.04 in the n=3 amplitude results in the design margin increasing from 13.6 - 18.1 percent to 23.6 - 29.0 percent.
- Residual stresses due to simulation of the introduction of the n=3 imperfection have an increasing effect on the collapse pressure. This means that the correction for residual stresses can be neglected, resulting in real (corrected, only  $\beta_{mat}=0.96$  and  $\gamma_{num}=0.95$ ) collapse pressures of 6.75 MPa, 6.11 MPa and 6.11 MPa respectively (errors -5.5 percent, -1.9 percent and -5.9 percent). It is obvious that, for analyses of the whole cylinders including the effect of residual stresses ( $P_{calc-coll}$  is 7.0 MPa), the real (corrected) values for the collapse pressures might easily become larger than the experimental collapse pressures ( $7.0 \cdot 0.96 \cdot 0.95$  is 6.38 MPa, experimental collapse pressure is 6.23 MPa). Probably the  $\gamma_{num}$  has to be increased for these type of analyses, e.g. to a value 0.9 in stead of 0.95.

- In section 4.3 it has been discussed that it is unlikely that the relatively large error as found between the experimental and the calculated collapse pressure for CYL 2 (7.5 percent) is caused by problems on the calculation side. Possibly measured imperfections are incorrect, or, due to the large sensitivity of the applied radial force with respect to one rotation of the bolts used for the introduction of the  $n=3$  imperfection, possibly damage was already present when starting with the (experimental) pressure loading phase (for bolts M20 x 2.5, about  $2/3$  rotation is sufficient to increase the radial forces from 3340 N (necessary to obtain the imperfection) to 4200 N (collapse)).
- As discussed with the sponsor of this project, damage during the introduction of the imperfection is not likely to occur (bolts M12 with fine pitch were used). The relatively large error as found for CYL 2 will be discussed in future, when comparing experimental and calculated collapse pressures for the test cylinders including a deck.

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# LIST OF SYMBOLS

$A_n$	Fourier coefficient (cosine term) as function of n	[mm]
$B_n$	Fourier coefficient (sine term) as function of n	[mm]
k	Number of waves in axial direction	
n	Number of waves in circumferential direction	
P	External hydrostatic pressure load	[MPa]
$P_{bosmar(mode(i))}$	Pressure as calculated by BOSOR5, BOSPOST or MARC	[MPa]
$P_{calc(mode(i))}$	Pressure as calculated with certain method	[MPa]
$P_{design}$	Required design pressure	[MPa]
$P_k$	Elastic (plastic) buckling pressure	[MPa]
$P_{real(mode(i))}$	Expected 'real' collapse pressure (calculated value corrected for uncertainties and inaccuracies)	[MPa]
$P_{req(mode(i))}$	Minimum required collapse pressure	[MPa]
$P_{start}$	Starting pressure for eigenvalue analysis	[MPa]
$P_{subhul}$	Pressure as calculated with SUBHUL	[MPa]
R	Radius	[mm]
t	Hull thickness	[mm]
W0	Initial amplitude of imperfection	[mm]
$W_{glob}$	Amplitude of global imperfection	[mm]
$W_{loc}$	Amplitude of local imperfection	[mm]
$W_{tilt}$	Amplitude of stiffener tilt imperfection	[mm]
$\alpha_{oper}$	Operational safety factor	
$\alpha_{mode(i)}$	Safety factor regarding a particular failure mode	
$\beta$	Uncertainty factor	
$\beta_{int(mode(i))}$	Uncertainty factor regarding interaction of modes	
$\beta_{mat}$	Uncertainty factor regarding material quality	
$\beta_{nonaxi(mode(i))}$	Uncertainty factor regarding influence of nonaxi-symmetrical parts	
$\beta_{res(mode(i))}$	Uncertainty factor regarding residual stresses	
$\beta_{shape(mode(i))}$	Uncertainty factor regarding imperfect initial shape	
$\beta_{weld}$	Uncertainty factor regarding welding deficiencies	
$\gamma$	Inaccuracy factor	
$\gamma_{analnum(mode(i))}$	Relative inaccuracy factor of analytical calculation with respect to numerical calculation	
$\gamma_{num(mode(i))}$	Inaccuracy factor regarding numerical calculation	
$\theta$	Circumferential coordinate	



Subscript mode(i):           crit/coll-axi/glob/loc/tilt-hull/stiffener

- First word denotes critical or collapse pressure.
- Second word denotes axisymmetrical state, global imperfection, local imperfection or stiffener tilt imperfection.
- Third word denotes critical or collapse pressure occurring in the hull or in the stiffener. If this denotes the hull, it might be followed by 'midbay' or 'frame', denoting the position midbay between the stiffeners or at the connection with the stiffeners.

Subscript mode(i):           elas/plas-glob/loc/tilt

- First word denotes elastic or plastic buckling.
- Second word denotes global, local or stiffener tilt buckling mode.

## 1. INTRODUCTION

During the past five years considerable progress has been made with regard to insight in the structural design and analysis of submarine pressure hulls, particularly as far as stability and plasticity are concerned (ref. [2], [3], [5] - [12]). In 1991 a design methodology has been developed (ref. [1]). This design methodology makes the knowledge and experience available to the designer in the form of design directives for the pressure hull.

For validation purposes cylinders which have been tested are analyzed according to the design methodology. Chapter 3 gives the results for three aluminium test cylinders as tested in Canada, as calculated according to stage 1 to 4 of the design methodology. Chapter 4 gives results for these cylinders with the measured imperfections, and finally chapter 5 lists the conclusions.

## 2. GEOMETRY AND MATERIAL OF THE CANADIAN TEST CYLINDERS

A total number of 5 aluminium cylinders have been tested. The third and fourth model includes a deck. These cylinders are not considered within this report. The cylinders without deck are referred to as CYL 1, CYL 2 and CYL 5. Geometry of the perfect test cylinders and material model are given in fig. 2.1. So the mean shell radius is 111.35 mm, shell thickness is 2.7 mm, stiffener spacing is 40 mm and the external webs have dimensions 8.0 x 5.5 mm. The material has a yield stress of 240 - 260 MPa, a Young's modulus of 71 GPa, a Poissons's ratio of 0.32 and a plastic modulus of 7 GPa with maximum stress 270 MPa. The end caps are made from FE510.

The perfect cylinders are created by machining a 520 mm long section of 254 mm diameter, 12.7 mm thick, drawn 6061-T6 aluminium tubing. Imperfections with  $n=3$  are applied using a heavy circular frame with three screws spaced at 120 degrees. This device applies force to three ring frames (ring numbers 5, 6 (central ring) and 7) at a time. Measured imperfections for the three test cylinders are given in appendix 1 to 3.

In ref. [13] and [14] collapse pressures are given for idealized imperfections (in a buckling mode) as calculated with ADINA-81 and ADINA-86. Due to a different shell element formulation differences occur up to 8 percent (ADINA-81 generally gives a larger collapse pressure). From chapter 5 of ref. [13], calculated collapse pressures for models with measured imperfections are 7.40 and 6.92 MPa for CYL 1 and CYL 2 respectively (ADINA-81). Experimental collapse pressures are 7.14 and 6.23 MPa. So the analyses overestimate the collapse pressures with 3.6 and 11.07 percent respectively. For CYL 5, ref. [15] gives a calculated collapse pressure (measured imperfections) of 6.85 MPa (ADINA-86), while the experimental collapse pressure for CYL 5 is 6.49 MPa.

### 3. RESULTS FOR THE CANADIAN TEST CYLINDERS ACCORDING TO THE DESIGN METHODOLOGY

#### 3.1 Additional remarks on the design methodology

This section gives some additional remarks on the design methodology (ref. [1]), considering uncertainty values and the choice of analytical methods which were not prescribed thus far. These data have to be chosen before the methodology can be used. Besides, some errors and incompletenesses as detected during the analyses are corrected.

- During stage 1 not only  $\beta_{mat}$  has to be determined, but also  $\beta_{weld}$ .
- Some unknown uncertainties have been chosen, namely:
  - $\beta_{res(elas-tilt)}$  has been set to 0.98, so equal to that for local and global buckling.
  - $\beta_{res(plas-loc)}$  and  $\beta_{res(plas-tilt)}$  have been set to 1.0.
  - $\beta_{res(coll-mode(i))}$  has been set to 0.95 for all modes, so equal to  $\beta_{res(coll-glob-stiffener)}$ .
  - $\beta_{nonaxi}$  has been set to 1.0 for all modes, because the cylinders do not have any nonaxisymmetrical structures.
- As discussed in appendix II.2 of the design methodology a value 0.9 has been used for  $\gamma_{num}$  for elastic global buckling modes and all plastic buckling modes. For all other modes  $\gamma_{num}$  is 0.95.
- It is not clear whether the amplitudes of the imperfections, which have to be determined during stage 1, have to be the amplitudes as found from the fourier analyses (as function of wave number  $n$ ) or the maximum amplitude with respect to the perfect radius (as function of the circumferential and axial coordinates, see appendix 1 to 3). Therefore for  $n=3$  both amplitudes are used for the imperfection analysis.
- Performing the calculations from ANAL-CALC-1 to 4 can lead to a collapse pressure in the hull midbay which is smaller than the corresponding critical pressure ( $P_{real}$  values). So the plastic reserve becomes negative. This leads to problems in the calculation of  $P_{real(coll-axi-hull)frame}$  (ANAL-CALC-3). Therefore this pressure has been taken equal to the corresponding critical pressure  $P_{real(crit-axi-hull)frame}$ .
- While performing the analyses as prescribed by ANAL-CALC-9, the infinity of the cylinder explicitly have to be defined in the model description, for example by giving the number of stiffeners a value 100, and the length between the bulkheads 100 times the stiffener spacing.
- For ANAL-CALC-10 and 11, no calculation method was prescribed. As for the

infinitely long cylinder, subroutine BS5500 has been chosen. For elastic buckling,  $\gamma_{\text{analnum}}$  is set to 0.95. For critical pressures in case of a global imperfection, both for the finite and the infinite cylinder,  $\gamma_{\text{analnum}}$  is set to:

- 0.98, for  $P_{\text{crit}} < 0.7 P_{\text{buck}}$
- 0.94, for  $0.7 P_{\text{buck}} < P_{\text{crit}} < 0.8 P_{\text{buck}}$
- 0.90, for  $P_{\text{crit}} > 0.8 P_{\text{buck}}$ .

- The requirements of condition 1 and 2 of stage 2.2 do not have to be satisfied for the global buckling pressures and the critical pressures in case of a global imperfection calculated for the infinitely long cylinder, unless for these modes (infinitely long) separate  $P_{\text{req}}$  were calculated during stage 1.
- This holds also for condition 1 and 2 of stage 3.
- Condition 4 of stage 3 has to be changed in:
  - $P_{\text{bosmar(plas-tilt)}} > 0.9 P_{\text{bosmar(coll-axi)}}$
  - $P_{\text{bosmar(plas-glob)}} > 0.9 P_{\text{bosmar(coll-axi)}}$
  - $P_{\text{bosmar(plas-loc)}} > 0.9 P_{\text{bosmar(coll-axi)}}$
- During the analyses of stage 4, it was found necessary to make a new BOSOR5 postprocessor, giving the axial stations, prebuckling solutions and buckling modes with more than only 3 significant digits. Therefore the remark from appendix III.4 of the design methodology considering the mesh sizing for the BOSPOST calculations is no longer valid. A doubled number of mesh points for the BOSPOST calculations is now recommended.
- The inaccuracy factor  $\gamma$  for the BOSPOST calculations consists of a  $\gamma_{\text{num}}$  only. No  $\gamma_{\text{analnum}}$  is defined. Because BOSPOST values are always an underestimation, this has not been changed.
- Some additional remarks considering the use of BOSPOST:
  - The reference surface of the BOSOR5 segments have to be in the middle of the physical segments.
  - For checking purposes, elastic buckling pressures can be calculated with BOSPOST, see ref. [7]. It is important to note that the part of the circumference taken into account within BOSPOST for the integration of energies depends on the minimum wave number of the two modes considered. For example, if  $n=4$  and  $n=5$  are used within one BOSPOST run, the circumference taken into account is 90 degrees, based on  $n=4$ . Of course, the elastic buckling pressure calculated for  $n=5$  will not correspond with the BOSOR5 value ( $n=5$  does not fit within 90 degrees). For calculation of critical pressures, this problem is not important.

### 3.2 Stage 1, canadian test cylinders

According to ref. [1], in stage 1 the design conditions have to be defined. Because the geometry of the cylinders is known, the specification side (fig. 1.2 of ref. [1]) can be skipped. So  $P_{design}$ ,  $\alpha_{oper}$ ,  $\alpha_{(mode\ i)}$  and  $P_{req(mode\ i)}$  don't have to be determined.

- Determine  $\beta_{mat}$  and  $\beta_{weld}$ :

According to ref. [13] the yield stress ranges from 240 to 260 MPa. Ref. [13] uses a yield stress of 250 MPa for the calculations. For comparison purposes of the analytical results, in this report a yield stress of 250 MPa is used, which results in  $\beta_{mat} = 240/250 = 0.96$ . Because the aluminium test cylinders are not welded,  $\beta_{weld}$  is set to 1.0.

- Determine  $W_{glob}$  for  $n=2$  to  $n=5$ ,  $W_{loc}$  for  $n=2$  to  $n=25$  and  $W_{tilt}$  for  $n=0$  to  $n=5$  or 6.

	Test cylinder CYL 1		Test cylinder CYL 2		Test cylinder CYL 5	
n	$W_{glob}$	Ring	$W_{glob}$	Ring	$W_{glob}$	Ring
2	0.08	5	0.10	6	0.20	4
3	0.42	6	0.68	6	0.60	6
4	0.042	5	0.060	6	0.065	6
5	0.025	7	0.033	7	0.025	5
n	$W_{loc}$	Ring	$W_{loc}$	Ring	$W_{loc}$	Ring
2-25	0.1	-	0.1	-	0.1	-
n	$W_{tilt}$	Ring	$W_{tilt}$	Ring	$W_{tilt}$	Ring
0-5	0.1	-	0.1	-	0.1	-
Max.	0.86	5	1.15	6	1.02	5

Table 3.1 Amplitude imperfections for n and maximum values in mm as taken from measured radii

For the determination of the amplitudes of the imperfections, the measured radii as delivered by the sponsor of the project and as given in appendix 1 to 3 are used. From these values the imperfection amplitudes can be found as given in Table 3.1. The maximum values from Table 3.1 are of course taken from the amplitudes with respect to the measured radius for  $n=0$ , because a small change in the perfectly round radius will hardly influence any of the  $P_{real(mode\ i)}$ .

### 3.3 Stage 2, canadian test cylinders

Because dimensions are known already, stage 2.1 of the design methodology can be skipped. Before the analyses as prescribed in stage 2.2 were carried out, the modelling of the web for the SUBHUL runs has been investigated. Two runs have been carried out, one with web 6 x 5.5 mm and flange 5.5 x 2 mm, and one with web 2 x 5.5 mm and flange 5.5 x 6 mm. SUBHUL results for both cases were exactly equal, except for the critical pressure of the flange as calculated by subroutine PULOS (values 11.669 and 11.474 MPa respectively).

The analyses as prescribed in stage 2.2 of the design methodology have been carried out with web 2 x 5.5 mm and flange 5.5 x 6 mm. Results are given in appendix 4. These results, together with results from other stages of the methodology are given in fig. 3.10 to 3.15, and will be discussed in section 3.6.

### 3.4 Stage 3, canadian test cylinders

Results as calculated with MARC, BOSOR5 and BOSPOST according to stage 3 of the design methodology are given in appendix 5. Models and calculation procedures are as described in appendix III of the design methodology.

For the MARC models, 8 elements are used in circumferential direction for half a wave. The circumference of the models depends on the wave number considered. For example, model n12 from Table A5.3 has a circumferential length of 15 degrees, so 15/8 degrees per element. Results are given in Table A5.3.

### 3.5 Stage 4, canadian test cylinders

Results are listed in Table A6.1 to A6.4 of appendix 6. Models and calculation procedures are as described in appendix III of the design

methodology. During this stage, it was found that elastic buckling pressures as calculated with BOSPOST for checking purposes (see ref. [7]), have a large error in comparison with the BOSOR5 values. Therefore, see section 3.1, a new BOSOR5 postprocessor has been made which gives the results with a sufficient number of digits. Now the three methods for the calculation of the derivatives as implemented in BOSPOST (C05NBF, E02BAF and E01BEF, see ref. [7]) correspond quite well with each other. Errors between elastic buckling pressures for the finite cylinder as calculated with BOSOR5 and BOSPOST are only a few percent.

Because the elastic buckling pressures for  $n=2$  and  $n=4$  are considerably larger than for  $n=3$ , while the imperfection amplitudes are considerably smaller, MARC analyses for a  $n=2$  and  $n=4$  imperfection are only performed for the largest imperfection amplitude (CYL 5).

Due to the absence of nonaxisymmetrical structures, stage 5 of the design methodology can be skipped.

### 3.6 Discussion of results

Fig. 3.1 to 3.9 show some typical deformed geometries and buckling modes. Fig. 3.2 shows that up to  $n=20$  the minimum buckling pressure corresponds to a global buckling mode. Fig. 3.3 shows that, starting from  $n=14$ , it is no longer possible to calculate the buckling pressure corresponding to a local buckling mode. Generally this is possible by suppressing the radial displacements during buckling at the connection with the web, but as fig. 3.3 c) shows, from  $n=14$  and larger this does not result in a local buckling mode. Fig. 3.4 gives some elastic buckling modes as calculated with MARC. The  $n=14$  buckling modes (3.4 e)) corresponds with BOSOR5 results (no radial displacements for the web for a global  $n=14$  buckling mode). Fig. 3.5 gives elastic buckling modes as calculated with BOSOR5 for the finite cylinder, and fig. 3.6 gives some plastic buckling modes. Fig. 3.7 gives displacements and Von Mises stresses as calculated by MARC for the axisymmetrical (perfect) finite cylinder. It is obvious that collapse occurs in the hull. Neither end caps nor webs yield already. Fig 3.8 gives the elastic buckling modes as calculated with MARC for the finite cylinders. Typical results for the imperfection analyses as carried out with MARC are given in fig. 3.9. Both with a  $n=2$  and a  $n=3$  imperfection, the end caps show small deformations according to the wave number. This effect has disappeared for a  $n=4$  imperfection (not shown in fig. 3.9).



Yielding of the end caps does not occur at pressure loads smaller than the collapse pressures.

Results considering the calculated pressure values, as given in the tables from appendix 4 to 6, are given in fig. 3.10 to 3.15. These figures contain all values from these appendices.

Fig. 3.10, giving pressure values of the axisymmetrical state, shows that:

- Values for the critical pressures of the hull, as calculated with different methods as prescribed by the design methodology, correspond quite well with each other. Errors are only a few percent, see fig. 3.10 a) and b).
- The same conclusion is valid for the critical pressures of the webs, see fig. 3.10 c).
- Numerical collapse pressures (fig. 3.10 d) correspond well with each other. Analytical collapse pressures show a larger difference.
- An important requirement of the design methodology is that the design as available after performing stage 2, should be such that large modifications during the other stages can be avoided. So the analytical pressures, corrected for inaccuracies and uncertainties ( $P_{real}$ ) should be the smallest values of all  $P_{real}$ . The numerical calculations as prescribed during stage 3 to 5 are used to reduce uncertainties and inaccuracies. Fig. 3.10 shows that for the axisymmetrical state this requirement is satisfied.
- Summarizing the results from fig. 3.10, critical pressures for the hull ( $P_{calc}$ -values) are 7.1 MPa (hull, midbay) and 6.3 MPa (hull, at web), resulting in a collapse pressure which ranges from 8.85 MPa (BOSOR5, finite) to 9.4 MPa (MARC, finite). For a discussion of this relatively large difference is referred to section 4.1.

Fig. 3.11, giving pressure values considering elastic and plastic buckling, shows that:

- The elastic local buckling pressures as calculated with BOSOR5 are a little larger than the MARC values (fig. 3.11 a). This is caused by the fact that the BOSOR5 local buckling pressures for these wave numbers were calculated including the suppression of the radial displacement at the connection between hull and web during buckling. (global buckling pressure is smaller than local buckling pressure). Starting from  $n=14$ ,

from which the extra boundary condition was no longer necessary, BOSOR5 and MARC results correspond quite well with each other.

- The error at the minimum local buckling pressure ( $n=8$ ) between MARC and analytical results is about 30 percent.
- Global buckling pressures (fig. 3.11 b) correspond quite well with each other. Only the analytical solution for  $n=4$  and 5 shows a larger error.
- As for the axisymmetrical state, minimum  $P_{real}$  are found with the analytical solution, except for the global buckling pressure for  $n=4$  and 5.
- Summarizing, calculated (numerical) minimum values are 33.6 MPa (local,  $n=8$ ), and 11.5 MPa ( $n=3$ , global). Indeed, plastic buckling pressures (fig. 3.11 d) are quite close to the plastic axisymmetrical collapse pressure.

Fig. 3.12, giving pressure values in case of a local or a tilt imperfection, shows that (MARC results for  $n=14$  were calculated for both the local and the global mode):

- MARC and BOSPOST results correspond quite well. Analytical results for small wave numbers are less reliable.
- For larger wave numbers, which are generally important for local imperfections,  $P_{real}$  values of the analytical solution meet the requirement of the design methodology as discussed before.
- Results from fig. 3.12 e) shows that critical and collapse pressures in case of a tilt imperfection ( $n=0$ ,  $w_0=0.1$  mm) are nearly equal to those of the axisymmetrical state.
- Summarizing, collapse pressures (fig. 3.12 d) remain larger than 8.5 MPa.

Fig. 3.13, giving pressure values in case of a global imperfection for the infinitely long cylinders, shows that:

- Analytical, BOSPOST and MARC results for critical pressures for the web (fig. 3.13 a) - c)) and for the hull midbay (3.13 d) - f)) correspond quite well with each other.
- Critical pressures for the hull at the connection with the web (fig. 3.13 g) - i)) as calculated with BOSPOST show differences with MARC values up to 10 percent.
- Apart from the  $n=2$  imperfection, smallest collapse pressures occur for the  $n=3$  imperfection with maximum amplitude (fig. 3.13 j)).
- Again, analytical calculated values meet the requirement of the design

methodology that the analytical  $P_{real}$  values have to be the smallest values.

Fig. 3.14, giving pressure values in case of a global imperfection for the finite cylinders as a function of the wave number, shows that:

- The correspondence of the critical pressures for the web (fig. 3.14 a) - c)) as calculated with BS5500 (analytical), BOSPOST and MARC is slightly worse than for the infinitely long cylinders. Best correspondence is obtained for the smallest critical pressure values.
- Again, the analytical calculated  $P_{real}$  are the smallest values.
- Critical pressures as calculated for the hull midbay (fig. 3.14 d) - f)) with BOSPOST and MARC can differ up to 10 percent.
- Critical pressures for the hull midbay (fig. 3.14 g) - i)) can differ up to 20 percent (about 0.8 MPa at pressure level of about 4 MPa). About 0.2 MPa of this error is due to the difference between BOSOR5 and MARC values as found in the critical pressures for the axisymmetrical case (see Table A6.1).
- Finally fig. 3.14 j) gives the collapse pressures as calculated with MARC. Smallest collapse pressures occur for the  $n=3$  imperfection, and are summarized in Table 3.2.

Finally, fig. 3.15 gives critical and collapse pressures in case of a global imperfection for the finite cylinders as a function of the ring

Test cylinder	Amplitude $n=3$	$P_{calc}$ MARC [MPa]	$P_{real}$ MARC [MPa]
CYL 1	0.42 mm	7.5	5.85
	0.86 mm (max)	6.5	5.07
CYL 2	0.68 mm	6.9	5.38
	1.15 mm (max)	6.1	4.76
CYL 5	0.60 mm	7.1	5.54
	1.02 mm (max)	6.3	4.91

Table 3.2 Calculated and real (corrected for uncertainties and inaccuracies) collapse pressures (MARC) for the finite cylinders with imperfections according to the  $n=3$  buckling mode

number. For  $n=2$  and  $n=4$  (fig. 3.15 g) and h)), carried out with the largest amplitude for these wave numbers (CYL 5), critical and collapse pressures are very close to each other, and close to those of the axisymmetrical state. So amplitudes for  $n=2$  and  $n=4$  from Table 2.1 are negligible small.

#### 4. CANADIAN TEST CYLINDERS WITH MEASURED IMPERFECTIONS

##### 4.1 MARC model and imperfections

To avoid the choice of boundary conditions for the cylinders with measured imperfections (imperfections are not symmetrical, see fig. 4.1 to 4.3), a MARC model has been made for the whole cylinder including the two end caps. Distributed pressure loads are present for hull and end caps. Of course, no reaction forces are present now, and therefore the convergence criterium as generally used (maximum value for maximum residual force divided by maximum reaction force) has to be changed. Convergence checking is done on displacements with accuracy 0.01. So if within one increment the maximum value of the change of the displacements (from present iteration to previous iteration) divided by the displacement of that increment is less than 0.01, the solution has converged. An important disadvantage is that for each increment, to be able to calculate the convergence criterium, at least two iterations are necessary.

At both poles of the end caps, all displacements and rotations are suppressed, except the axial displacements. To prevent rigid body modes, tangential displacements are suppressed at the nodes at the outer radius of both end caps, for a circumferential coordinate of 0 degrees, and the axial displacement is suppressed at a node in the hull at the connection with ring 6, also for a circumferential coordinate of 0 degrees.

The imperfection values from appendix 1 to 3 are linear interpolated, both in axial and circumferential direction. Results are given in fig. 4.1 to 4.3. Maximum values (fig. 4.1 c), 4.2 b) and 4.3 c)) correspond with the maximum amplitudes of the  $n=3$  imperfections as used in the foregoing chapter. Maximum values occur at 0, 135 and 240 degrees. Because the element length in circumferential direction is 10 degrees (6 elements for half a wave with  $n=3$ ), during interpolation the maximum amplitude for the second cylinder has been missed, see fig. 4.2 a). Because:

- the sponsor of this project noted that the measured radii for either CYL 1 or CYL 2 were probably rotated one position (so 15 degrees),
- the maximum for the second cylinder has to be included in the model,
- imperfections were introduced with a heavy circular frame with three screws spaced at 120 degrees,

the imperfection for CYL 2 has been rotated such that the maximum amplitude occurs at 120 degrees, see fig. 4.2 b).

Summing the amplitudes of fig. 4.1 a) and 4.3 a) (n=0 imperfection with respect to the radius 120.7 mm) with the amplitudes of fig. 4.1 c) and 4.3 c) (amplitudes with respect to the measured radius for n=0) results in the amplitudes of fig. 4.1 b) and 4.3 b) (amplitudes with respect to the radius 120.7 mm). Remarkable is that CYL 1 has an outwards n=0 imperfection with respect to R=120.7 mm, which is not as expected considering the way in which the imperfections are introduced. However, the small n=0 imperfection will hardly influence the collapse pressures. Radial imperfections with respect to R=120.7 mm as given in fig. 4.1 b), 4.2 b) and 4.3 b) are added to the perfect model for the calculation of the critical and collapse pressures of the test cylinders with measured imperfections.

A summary of the maximum amplitudes is given in Table 4.1.

Test	Ampl. w.r.t. R n=0	Ampl. w.r.t. R=120.7 mm	direction	ring	degrees
CYL 1	0.86	1.30	inwards	5	0
CYL 2	1.15 1)	1.15 1)	inwards	6	120
CYL 5	1.02	1.25	inwards	5	240
1) n=0 term unknown					

Table 4.1 Summary of maximum measured amplitudes and positions

Due to computer limitations, the MARC model has less nodes (8966) and elements (9000) than the models from the foregoing chapter (for each stiffener spacing). To investigate this effect, first two runs were made in which the elastic buckling pressures and axisymmetrical critical and collapse pressures have been calculated. Elastic buckling pressures are calculated starting from a pressure load of 10 MPa. Results are given in Table 4.2. Some buckling modes are given in fig. 4.4. For each wave number two buckling modes are found with exactly the same buckling pressure. These buckling modes are equal, but the maximum amplitudes are rotated (in circumferential direction). The difference in the global n=3 buckling pressures for both MARC models is 3.5 percent. Differences increase with increasing wave number, due to the fact that for larger wave numbers less elements are available to describe half a wave of the buckling mode.

n	Pk MARC (whole model) [MPa]	Pk MARC (stage 4) [MPa]	Pk BOSOR5 (stage 4) [MPa]
2	16.25	16.27	16.7
3	12.03	11.62	11.5
4	17.78	16.79	16.4
4	19.32 (k=1)	22.30 (k=1.5)	-
All MARC buckling pressures already corrected for live load			

Table 4.2 Comparison of global elastic buckling pressures as calculated with MARC for the present model with those from stage 4

Results for the axisymmetrical critical and collapse pressures are given in Table 4.3. The only critical pressure showing a large difference with those from Table A6.1 (axisymmetrical MARC results, stage 4, finite cylinder), is that between the last ring and the end cap. This is the region with a decreasing thickness (from  $t=12$  mm at end cap to  $t=2.7$  mm). Because the present model has less elements for each stiffener spacing, the description of this region is less accurate. It is noted that critical pressures as calculated with the present model are symmetrical around ring number 6.

The collapse pressure of 8.8 MPa is a remarkable 0.6 MPa smaller than the axisymmetrical MARC result from Table A6.1. The present collapse pressure occurs due to a non-positive definite matrix, so plastic buckling occurs. This pressure corresponds quite well with the plastic buckling pressures as calculated with BOSOR5 (Table A6.2, 8.73 MPa for  $n=3$ ) and with the collapse pressures as calculated with MARC for the finite cylinder with a  $n=2$  or a  $n=4$  imperfection (Table A6.4). These two collapse pressures were also caused by plastic buckling, while the collapse pressures for a  $n=3$  imperfection from Table A6.4 were caused by failure to converge. So the only disagreement is found in the collapse pressures of the axisymmetrical state (8.8 MPa, plastic buckling, present model and 9.4 MPa, plastic buckling, Table A6.1). Considering fig. 4.5 (pressure - displacements curves nearly horizontal) and comparing fig. 4.6 with fig. 3.7 b) (Von Mises stresses quite close to each other), it is obvious that the collapse pressure of the present model is quite reliable. The difference can be caused by:

- the fact that plastic buckling for the model from Table A6.1 is only possible for a relatively large wave number, due to the circumferential length.
- the convergence criteria used. For the model from Table A6.1 the convergence criterium used is based on the maximum value for the maximum residual force divided by the maximum reaction force, which should be smaller than 0.01. For the present model, the maximum displacements change within one increment should be smaller than 0.01 (1 percent). Possibly, the first criterium allows larger errors than the second criterium, which might result in a larger collapse pressure.

So it can be concluded that the present mesh distribution is accurate, giving results which correspond with those from stage 4.

Position	P-MARC	Position	P-MARC
Midbay 6/7	7.0	At ring 6	6.4
Midbay 7/8	7.0	At ring 7	6.4
Midbay 8/9	7.0	At ring 8	6.4
Midbay 9/10	7.0	At ring 9	6.4
Midbay 10/11	6.8 1)	At ring 10	6.4
Midbay 11/12 (12= end cap)	8.2 2)	At ring 11	8.0 3)
Collapse pressure:			8.8
1): 0.1 MPa larger than MARC results from stage 4 (Table A6.1) 2): 1.2 MPa larger than MARC results from stage 4 (Table A6.1) This is the region with decreasing hull thickness. 3): 0.1 MPa smaller than MARC results from stage 4 (Table A6.1)			

Table 4.3 Critical and collapse pressures in MPa as calculated with MARC for a model of the whole cylinder, axisymmetrical state

#### 4.2 MARC results for cylinders with measured imperfections

MARC results considering the critical and collapse pressures for the three testcylinders with measured imperfections are given in Table 4.4. Only the two minimum critical pressures in circumferential direction for the axial



Position	$\theta$ (deg)	CYL 1	CYL 2	CYL 5
Hull, midbay 5/6	0.0	5.4	5.1	-
	120.0	-	5.0	5.2
	240.0	5.4	-	5.1
Hull, midbay 6/7	0.0	5.6	5.3	-
	120.0	-	5.2	5.2
	240.0	5.4	-	5.1
Hull, at ring 5	0.0	4.9	4.5	-
	120.0	-	4.4 **)	4.7
	240.0	5.0	-	4.5
Hull, at ring 6	0.0	4.8 **)	4.5	-
	120.0	-	4.5	4.6
	240.0	4.8 **)	-	4.4 **)
Hull, at ring 7	0.0	5.2	4.7	-
	120.0	-	4.6	4.5
	240.0	5.1	-	4.5
Web, ring 5	0.0	6.0	5.1	-
	120.0	-	4.9	5.3
	240.0	6.2	-	5.0
Web, ring 6	0.0	6.0	5.2	-
	120.0	-	4.7 *)	5.1
	240.0	5.9 *)	-	4.9 *)
Web, ring 7	0.0	6.5	5.4	-
	120.0	-	5.3	5.1
	240.0	6.2	-	5.1
Collapse pressure 1):		7.3 - 7.4	6.6 - 6.7	6.6 - 6.7
1) first pressure correct solution, second pressure failure to converge *) Minimum critical pressure for the webs **) absolute minimum critical pressure				

Table 4.4 Critical and collapse pressures in MPa as calculated with MARC for models of the whole cylinder with measured imperfections

positions from Table 4.4 are given. The absolute minimum is always included. Plots showing the displacements versus the external hydrostatic pressure are given in fig. 4.7 to 4.10. Fig. 4.11 to 4.13 give coloured contour plots of deformed geometries and Von Mises stresses at pressure loads just before collapse. Comparing the critical pressures from Table 4.4 with the displacements from fig. 4.7 to 4.10, shows a correct correspondence. Minimum critical pressures of the circumference correspond with maximum radial displacements.

An important question to be answered concerns the correspondence between calculated collapse pressures and experimental collapse pressures. These collapse pressures, also for the idealized imperfections in the  $n=3$  buckling mode (Table 3.2 and A6.4), together with the corrected values for inaccuracies and uncertainties according to the design methodology, are summarized in Table 4.5.

From these results, it is obvious that the calculated collapse pressures for the measured imperfections are always between the collapse pressures as calculated for an idealized  $n=3$  imperfection with amplitude equal to the  $n=3$  Fourier component and amplitude equal to the maximum amplitude as found on the circumference of the model. Of course, a  $n=3$  imperfection with amplitude equal to the Fourier term for  $n=3$  will overestimate the collapse pressure (amplitudes for other wave numbers neglected) and a  $n=3$  imperfection with amplitude equal to the maximum amplitude as measured somewhere on the cylinders will underestimate the collapse pressure (largest amplitude supposed to be present in the most dangerous imperfection only). Errors from ADINA results are equal to or larger than errors from the MARC results.

The errors from the last column of Table 4.5 show that an important aim of the design methodology, namely to reduce the design margin (see fig. 1.2 of ref. [1]), has been met. The real expected collapse pressure is only 11.9 to 7.4 percent smaller than the experimental collapse pressure, and is never larger than the experimental collapse pressure. However, it is obvious that the design margin which can be reached (which has to be as small as possible for an optimum design) is strongly dependent from the knowledge of the amount of imperfections.

Test CYL 1				
Experimental	7.14			
Type of calculation	P <sub>calc</sub>	Error [%]	P <sub>real</sub>	Error [%]
ADINA 81/86	7.4	3.6	-	-
Measured imperfection	7.4	3.6	6.41	-11.4
n=3, W0=0.42 mm (Fourier)	7.5	5.0	5.85	-22.1
n=3, W0=0.86 mm (maximum)	6.5	-9.8	5.07	-40.8
Test CYL 2				
Experimental	6.23			
Type of calculation	P <sub>calc</sub>	Error [%]	P <sub>real</sub>	Error [%]
ADINA 81/86	6.92	11.1	-	-
Measured imperfection	6.7	7.5	5.80	-7.4
n=3, W0=0.68 mm (Fourier)	6.9	10.8	5.38	-15.8
n=3, W0=1.15 mm (maximum)	6.1	-2.1	4.76	-30.9
Test CYL 5				
Experimental	6.49			
Type of calculation	P <sub>calc</sub>	Error [%]	P <sub>real</sub>	Error [%]
ADINA 81/86	6.85	5.5	-	-
Measured imperfection	6.7	3.2	5.80	-11.9
n=3, W0=0.60 mm (Fourier)	7.1	9.4	5.54	-17.1
n=3, W0=1.02 mm (maximum)	6.3	-3.0	4.91	-32.2
<p>Accuracy in calculated collapse pressures is +0.0, -0.1 MPa  n=3 Fourier is amplitude from Fourier analysis  n=3 maximum is maximum amplitude of circumference  Errors are with respect to experimental pressure:  positive value means overestimation  negative error means underestimation  For measured imperfections, P<sub>real</sub> calculated with:  <math>\beta_{mat} = 0.96</math>, <math>\beta_{res} = 0.95</math>, <math>\gamma_{num} = 0.95</math>  For other imperfections, P<sub>real</sub> calculated with:  <math>\beta_{mat} = 0.96</math>, <math>\beta_{res} = 0.95</math>, <math>\beta_{int} = 0.9</math>, <math>\gamma_{num} = 0.95</math></p>				

Table 4.5 Summary of experimental, calculated and corrected collapse pressures in MPa for collapse with n=3

Finally, fig. 4.14 shows critical and collapse pressures (calculated and real values) for all analyses with a  $n=3$  imperfection. Fig. 4.15 shows the quotients of these values with respect to the experimental collapse pressure or the calculated critical pressure in case of a measured imperfection. For a quotient close to 1.0, the correspondence is quite good, a quotient larger than 1.0 means an overestimation. These figures show that:

- accuracy increases with increasing analysis method (value closer to 1.0), except for the analytical solution for a finite compartment (analysis method 4).
- None of the real pressures (corrected values according to the design methodology) has a quotient larger than 1.0, so all real pressures are underestimations.
- Of course, the pressure values are strongly dependent of the amplitude of the imperfection. Considering the collapse pressures, quotients are:

1.05 - 1.11 - 1.09,	calculated values with smallest amplitude as based on Fourier analysis, for CYL 1, CYL 2 and CYL 5 respectively.
0.82 - 0.86 - 0.85,	real (corrected) values with smallest amplitude as based on Fourier analysis, for CYL 1, CYL 2 and CYL 5 respectively.
0.91 - 0.98 - 0.97,	calculated values with largest amplitude as found on the circumference, for CYL 1, CYL 2 and CYL 5 respectively.
0.71 - 0.76 - 0.76,	real (corrected) values with largest amplitude as found on the circumference, for CYL 1, CYL 2 and CYL 5 respectively.

So real values for the smallest amplitude show underestimations up to 18 percent, and real values for the largest amplitude show underestimations up to 29 percent. Considering the real values as obtained for the cases with measured imperfections (underestimations 11.39, 7.41 and 11.90 percent), it is obvious that the reduction of the difference between  $P_{real}$  and  $P_{req}$ , which was one of the aims of the design methodology (see fig. 1.3 of ref. [1]), is strongly dependent from the knowledge considering type and amplitude of the real imperfection.

- It is noted here that the residual stresses (see next section) have an increasing effect now on the collapse pressure. So the real values of the

collapse pressures are in fact corrected for residual stresses in the wrong direction. For the quotients above, this means that the real values can be divided by 0.95 (than values are obtained without any correction for residual stresses) resulting in values closer to 1.0 (but still smaller than 1.0).

#### 4.3 MARC results for CYL 2 including simulation of introduction of imperfection

Because of the relatively large difference as found between calculations and experiment for test CYL 2 (also with ADINA), some analyses have been carried out to investigate the effect of residual stresses which will occur during the introduction of the n=3 imperfection on the perfect test cylinder. The same mesh sizing has been used, for a model with six stiffener spacings and 60 degrees in circumferential direction. The sixth web has half the thickness, with symmetry conditions.

From fig. 4.2 b), some requirements for the final imperfection are (taken from the section of 60 to 120 degrees):

- the positive (outwards) radial amplitude of ring 6 has to be about 50 percent of the negative (inwards) amplitude of ring 6.
- the negative amplitude of ring 3 has to be 50 to 60 percent of the negative amplitude of ring 6.
- the negative amplitude of ring 2 has to be 20 to 25 percent of the negative amplitude of ring 6.

Radial inwards forces are applied at the outer radius of the webs at  $\theta=0$  degrees. These forces are increased to a maximum value and then reduced to zero. Of course, at the maximum value plasticity has to occur to get a resulting imperfection.

Fig. 4.16 shows the deformed geometries (so imperfections) when the forces have turned to zero, for forces applied at ring 6 and 5 (first deformed geometry), and for forces applied at ring 6, 5 and 4 (second deformed geometry). Comparing fig. 4.16 with fig. 4.2 b), both imperfections are rejected because they are too flat in axial direction.

Therefore forces are applied at four rings, e.g. ring 6 to 3. This corresponds more or less with the introduction of the imperfection on the

test cylinder (ring with three screws, applying radial forces at three webs, moved two webs in axial direction). The maximum radial forces are 3340 N for ring 5 to 3 and 1670 N for ring 6 (half web thickness). Deformed geometries at maximum forces and at zero forces are shown in fig. 4.17. This imperfection is supposed to be a correct description of the section from 60 to 120 degrees of the measured imperfection (fig. 4.2 b)), with equal maximum negative (inwards) amplitude.

Fig. 4.18 shows the residual stresses corresponding with the final imperfection from fig. 4.17. Residual maximum Von Mises stresses in the end cap are 20.1 MPa for inside and outside and 15.4 MPa for the mid layer. Maximum residual Von Mises stresses for the webs decreases from 276.8 MPa for layer 1 to 275.9 MPa for layer 11 (decreasing with about 0.1 MPa per layer). So, as is clear from fig. 4.18 c) and d), residual stresses in the webs are mainly membrane stresses. Maximum residual Von Mises stresses for the hull, for layer 1 (inside) to layer 11 (outside) are 232.9, 178.2, 126.9, 78.8, 53.6, 70.5, 107.5, 152.7, 194.3, 220.1 and 230.1 MPa respectively. These residual stresses consist mainly of bending and are very local, see fig. 4.18 a) and b). Fig. 4.19 shows some radial displacements as a function of increasing and decreasing radial forces. The remaining values at the end of the decreasing force path are the amplitudes of the imperfection.

The model from fig. 4.17 including the imperfection and the residual stresses from fig. 4.18 has been loaded with hydrostatic pressure. Collapse (failure to converge) occurs at 7.0 MPa, which is 0.3 MPa larger than the collapse pressure for CYL 2 with measured imperfection, but without residual stresses. Opposite to the model of the whole cylinders, this analysis has been carried out using the convergence criterium based on residual forces, see section 4.1. To investigate this effect, the same analysis has been carried out using the convergence criterium based on displacements. The same collapse pressure of 7.0 MPa was found. So residual stresses seems to have an increasing effect on the collapse pressure.

Fig. 4.20 and 4.21 give radial displacements, and compare these displacements with those from the analysis of CYL 2. Fig. 4.22 gives the total stresses (residual stresses due to imperfection and stresses due to pressure loading) at a pressure of 6.8 MPa, so just before collapse. Comparing these stresses with those from CYL 2 (fig. 4.12), collapse seems to occur in the hull now. Fig. 4.23 compares Von Mises stresses in the web

at the maximum inwards radial displacement from CYL 2 ( $\theta=120$  degrees) and from the present analysis ( $\theta=0$  degrees), as a function of the pressure, for the nodes at different radii. Without residual stresses (fig. 4.23 a)), yielding starts at the outer radius, and just before collapse all nodes, except the node closest to the hull, are yielding. With residual stresses (fig. 4.23 b)), the Von Mises stresses first decrease for the nodes with larger radii, resulting in yielding to start at nodes close to the hull (small radii). Of course, tangential bending stresses are smaller for these nodes than for the outer radius, resulting in a larger collapse pressure. Once more this is illustrated by fig. 4.24, showing both tangential and Von Mises stresses as a function of radial force and pressure load, as a function of the radius. These figures show that:

- stresses in the webs are mainly determined by the tangential stress component (Von Mises stress nearly equal to tangential stress).
- for three nodes at larger radii (fig. 4.24 a) to c), radii 120.7 to 117.583 mm) large residual tensile tangential stresses occur.
- because without residual stresses collapse is introduced here by large tangential compressive stresses, the tensile residual tangential stresses causes the collapse pressure to increase.

Because the imperfection as obtained with the radial forces is not exactly equal to the measured imperfection (fig. 4.2 b)), some analyses have been carried out to calculate collapse pressures which can be compared to obtain the influence of residual stresses:

- an analysis has been carried out using the deformed geometry as obtained with the radial forces (fig. 4.17, second deformed geometry) as an imperfection (added to perfect model). So no residual stresses are present. The resulting collapse pressure is 6.3 MPa.
- an analysis has been carried out using the measured imperfection from CYL 2, from  $\theta=60$  to  $\theta=120$  degrees (fig. 4.2 b)), as an imperfection. Comparing this collapse pressure with the collapse pressure from CYL 2 gives the effect of the symmetry conditions. The collapse pressure as found with this model is 6.4 MPa. Comparing critical pressures with those from the model with the simulated imperfection, shows that:
  - Critical pressures for the inside of the hull at the connection with the rings are about 0.2 MPa smaller.
  - Smallest critical pressure for the outside of the hull midway between

the rings (midway ring 5 and 6) is equal. Critical pressures for the other midbay positions are 0.1 to 0.2 MPa smaller, which is of course caused by the difference in the imperfections (simulated imperfection has smaller amplitudes for rings unequal to 6 than the measured imperfection).

- Critical pressures for web 6 are equal. Critical pressures for the other rings differ at most 0.1 MPa.

So indeed the simulated imperfection is a correct description of the measured imperfection.

A comparison of critical pressures as calculated for the whole cylinder with measured imperfection (CYL 2) and those from the model with 60 degrees in circumferential direction, measured imperfection (from 60 to 120 degrees, 6 left rings from the lowest figure of fig. 4.2 b)), and symmetry conditions, shows that all critical pressures are 0.2 to 0.3 MPa smaller, resulting in a collapse pressure of 6.4 MPa versus 6.7 MPa. This is obvious considering that the worst section of the measured imperfection has been chosen (compare radial imperfection of ring 5 to 3 with ring 7 to 9, fig. 4.2 b), (in figure below of fig. 4.2 b), ring 1 is located at the right side, ring 11 at left side)). As known from previous research, the collapse pressure can increase if stiffeners next to the critical stiffener are able to support the most critical stiffener. Besides, probably the symmetry conditions have a decreasing effect on the critical and collapse pressures (extra requirements for displacement field).

#### Summarizing:

- Residual stresses due to the introduction of the n=3 imperfection have an increasing effect on the collapse pressure of 0.7 MPa (from 6.3 to 7.0, for a model covering one twelfth of the test cylinder).
- The simulated imperfection corresponds quite well with the measured imperfection (ring 1 to 6, section 60 to 120 degrees), resulting in nearly equal collapse pressures.

From the results above it is obvious that residual stresses can not be responsible for the relatively large difference as found between the calculated collapse pressure (MARC and ADINA) and the experimental collapse pressure for CYL 2.



In search for an explanation of the difference, the analyses from Table 4.6 have been carried out. These results are a check of the results of the whole model. The mean value of the collapse pressures is 6.69 MPa, so equal to the collapse pressure of the whole model. The collapse pressures from Table 4.6 shows that the criteria used for sorting the imperfections from most severe to less severe, are not quite correct. Therefore Table 4.7 give the same results, but now the imperfections are sorted on another parameter. For the nodes in the section considered, NAG-routine E04ABF (minimizing a function) and E04FDF (minimizing a least square problem) are used to minimize the sum of errors in square between W0 times the radial

Ring no.	Section (deg)	Max. ampl. inwards [mm]	2nd ampl. inwards [mm]	Max. ampl. outwards [mm]	P <sub>coll</sub> [MPa]
1 - 6	60 - 120	-1.146	-1.045	0.657	6.4
1 - 6	120 - 180	-1.146	-1.045	0.581	6.4
6 - 11	60 - 120	-1.146	-0.630	0.657	6.5
6 - 11	120 - 180	-1.146	-0.630	0.581	6.7
1 - 6	0 - 60	-0.994	-0.892	0.657	6.5
1 - 6	300 - 360	-0.994	-0.892	0.492	6.7
6 - 11	0 - 60	-0.892	-0.744	0.657	6.6
6 - 11	300 - 360	-0.892	-0.719	0.492	6.8
6 - 11	180 - 240	-0.719	-0.664	0.581	6.9
6 - 11	240 - 300	-0.719	-0.664	0.492	7.0
1 - 6	180 - 240	-0.689	-0.664	0.581	6.9
1 - 6	240 - 300	-0.689	-0.664	0.492	6.9
Sections sorted using as criteria:					
1) max. negative amplitude as measured for ring 5 to 7.					
2) 2nd max. negative amplitude as measured for ring 5 to 7.					
3) max. positive amplitude as measured for ring 5 to 7.					

Table 4.6 Collapse pressures as a function of amplitudes, calculated for the different sections of the measured imperfection (CYL 2)

amplitude of the n=3 buckling mode and the measured radial imperfection with respect to the radius for n=0. So:

minimize  $\Sigma (w_{\text{eig. } n=3} * W_0 - w_{\text{meas. w.r.t. } Rn=0})^{**2}$ , with  $\Sigma$  for all nodes of the section.

Both NAG-routines (E04ABF used as check) deliver the same results. The idea is that the better the measured imperfection fits on the n=3 buckling mode, the more dangerous that imperfection will be, resulting in a smaller collapse pressure. The collapse pressures from Table 4.7 show a good correlation with the thus calculated  $W_0$  for n=3. If  $W_0$  is nearly equal (first and last three rows), the collapse pressure increases with a decreasing residual sum of squares. The same procedure has been carried out for CYL 1, CYL 2 and CYL 5, using the imperfection amplitudes of all nodes at the outer radius of the rings. Results are given in Table 4.8.

Ring no.	Section (deg)	$W_0$ calc. with least square method	Residual sum of squares	$P_{\text{coll}}$ [MPa]
1 - 6	120 - 180	0.81924	1.31065	6.4
1 - 6	0 - 60	0.81232	0.47960	6.5
1 - 6	60 - 120	0.80585	1.57916	6.4
6 - 11	60 - 120	0.75531	1.09345	6.5
6 - 11	0 - 60	0.75079	0.28874	6.6
6 - 11	120 - 180	0.74149	0.95557	6.7
1 - 6	300 - 360	0.65156	0.97764	6.7
6 - 11	300 - 360	0.61043	0.70354	6.8
1 - 6	240 - 300	0.59511	0.29608	6.9
6 - 11	240 - 300	0.56319	0.27929	7.0
1 - 6	180 - 240	0.56116	1.17010	6.9
6 - 11	180 - 240	0.56043	0.88640	6.9

Table 4.7 Collapse pressures as a function of amplitudes in n=3, as calculated according to the least square method, for CYL 2

From the discussion of the results from Table 4.7, the W0 values and the residual sums of squares from Table 4.8 lead to the conclusion that the collapse pressure from CYL 2 is expected to be larger than the collapse pressure from CYL 5! Summarizing, it can be concluded that:

- a reliable relationship has been found between the amplitude for a n=3 imperfection as calculated with the least square method and the collapse pressure, for the twelve sections from CYL 2 with measured imperfection, see Table 4.7.
- applying this relationship to results for the whole cylinders (Table 4.8) leads to the conclusion that the collapse pressure of CYL 2 is expected to be larger than the collapse pressure of CYL 5.
- this is confirmed by the calculated collapse pressures from both ADINA (6.92 and 6.85 MPa) and MARC (6.7 and 6.7 MPa), but certainly not by the experimental collapse pressures (6.23 and 6.49 MPa).

Therefore it seems to be justified to suppose that calculated values are correct for the given measured imperfection.

	W0 calc. with least square method	Residual sum of squares	Mean error per node 1)	P <sub>coll-exp.</sub> [MPa]	P <sub>coll-calc</sub> [MPa]
CYL 1	0.34079	3.0066	0.087	7.14	7.4
CYL 2	0.65114	7.5920	0.138	6.23	6.7
CYL 5	0.69057	9.8156	0.157	6.49	6.7
1): Residual sum of squares divided by number of nodes (396), and square root taken					

Table 4.8 N=3 amplitudes as calculated with the least square method

Another reason for the large difference as found between the experimental and the calculated collapse pressure for CYL 2, might be that collapse has been started during the introduction of the n=3 imperfection (CYL 2 having the largest amplitude). To investigate this, an analysis has been carried out as described before for the investigation of the influence of residual stresses, but now the radial forces on the rings (3 to 6) are increased until collapse occurs. Collapse occurs at a radial force per ring of 4200

N. So the forces necessary to obtain the measured imperfection (3340 N per ring) are equal to 80 percent of the forces for which collapse occurs.

According to fig. 4.19, at the end of the increasing radial force path, an increase of 1.0 mm in the radial displacements corresponds with an increase in the radial forces of about 500 N. For the increase in the forces from 3340 to 4200 N, the corresponding increase in the radial displacement is 1.72 mm. From fig. 3 of ref. [4], an estimation can be made of the size of the bolts used in the heavy circular frame to introduce the  $n=3$  imperfection. Suppose that bolts M20 x 2.5 are used, then a  $2/3$  rotation of the bolts is sufficient to increase the radial forces from 3340 N (necessary to obtain the imperfection) to 4200 N (collapse). Considering this sensitivity, it might quite well be possible that test CYL 2 had already some damage at the end of the introduction of the imperfection, resulting in the smaller collapse pressure.

## 5. CONCLUSIONS

- First, in section 3.1 additional remarks are given on the design methodology, necessary to be able to use the design methodology. Some uncertainties and inaccuracies are filled, which are used within this report.
- Fig. 3.10 to 3.15, show that an important aim of the design methodology has been met, namely that real pressure values obtained with analytical methods for the different failure modes are the smallest ones (except for some uninteresting values as elastic global buckling for  $n=4$  and larger, and critical pressures for the hull in case of a local imperfection for  $n=7$  and smaller). So continuing from stage 2 to stage 3 - 5 of the design methodology, no modifications are necessary unless the weight has to be reduced. Fig. 4.14 and 4.15 show the increasing accuracy using more sophisticated analysis methods.
- Errors for CYL 1, CYL 2 and CYL 5 with respect to experimental collapse pressures are (in percents, positive is overestimation of collapse pressure, results for  $n=3$  imperfections for finite cylinder from Table A6.4):

3.6 /	7.5 /	3.2	calculated collapse pressures for measured imperfections.
-10.2 /	-6.9 /	-10.6	real (corrected) collapse pressures for measured imperfections.
5.0 /	10.8 /	9.4	calculated collapse pressures for smallest amplitude (Fourier analysis) for $n=3$ imperfections.
-18.1 /	-13.6 /	-14.6	real (corrected) collapse pressures for smallest amplitude (Fourier analysis) for $n=3$ imperfections.
-9.0 /	-2.1 /	-2.9	calculated collapse pressures for largest amplitude (on circumference) for $n=3$ imperfections.
-29.0 /	-23.6 /	-24.3	real (corrected) collapse pressures for largest amplitude (on circumference) for $n=3$ imperfections.

It is obvious that calculated collapse pressures will result in an overestimation, unless the maximum amplitude as found on the circumference is supposed to be present in the worst buckling mode. Of course, this will always result in an underestimation. Using the Fourier term for  $n=3$  as amplitude for the  $n=3$  imperfection, and correcting these

collapse pressures for remaining uncertainties and inaccuracies, results in an underestimation (design margin, see ref. [1]) of 13.6 to 18.1 percent. Using the measured imperfections (more sophisticated analyses, reduction of uncertainties) results in a design margin of only 6.9 to 10.6 percent.

- Although the design margins above are sufficiently small, which was one of the aims of the design methodology, the amount of reduction of the design margin is strongly dependent of the knowledge of type and amplitude of the imperfection. As the results above show, a factor 1.69 to 2.04 in the  $n=3$  amplitude results in the design margin increasing from 13.6 - 18.1 percent to 23.6 - 29.0 percent.
- If residual stresses are taken into account, the calculated collapse pressures will increase (with at most 0.7 MPa, residual stresses will be smaller at smaller amplitudes). This means that the correction for residual stresses can be neglected, resulting in real (corrected, only  $\beta_{mat}=0.96$  and  $\gamma_{num}=0.95$ ) collapse pressures of 6.75 MPa, 6.11 MPa and 6.11 MPa respectively (errors -5.5 percent, -1.9 percent and -5.9 percent). It is obvious that, for analyses of the whole cylinders including the effect of residual stresses ( $P_{calc-coll}$  is 7.0 MPa), the real (corrected) values for the collapse pressures might easily become larger than the experimental collapse pressures ( $7.0 \cdot 0.96 \cdot 0.95$  is 6.38 MPa, experimental collapse pressure is 6.23 MPa). Probably the  $\gamma_{num}$  has to be increased for these type of analyses, e.g. to a value 0.9 in stead of 0.95.
- In section 4.3 it has been discussed that it is unlikely that the relatively large error as found between the experimental and the calculated collapse pressure for CYL 2 is caused by problems on the calculation side. Possibly measured imperfections are incorrect, or, due to the large sensitivity of the applied radial force with respect to one rotation of the bolts used for the introduction of the  $n=3$  imperfection, possibly damage was already present when starting with the (experimental) pressure loading phase (for bolts M20 x 2.5, about 2/3 rotation is sufficient to increase the radial forces from 3340 N (necessary to obtain the imperfection) to 4200 N (collapse)).
- As discussed with the sponsor of this project, damage during the introduction of the imperfection is not likely to occur (bolts M12 with

fine pitch were used). The relatively large error as found for CYL 2 will be discussed in future, when comparing experimental and calculated collapse pressures for the test cylinders including a deck.

#### REFERENCES

- [1] J.E. van Aanhold and W. Trouwborst  
Directives for the structural design of submarine pressure hulls  
TNO-report B-91-1143, 1991
  
- [2] H.A. Lupker  
Toepassing van de formule van Kendrick op het middenschip van de OZB-Walrus klasse  
TNO-report 5073731-86-1, 1986
  
- [3] H.A. Lupker  
Methode ter bepaling van de critische druk en plastische reserve voor een oneindig lange ringverstijfde cilinder  
TNO-report 5073838-87-1, 1987
  
- [4] N.G. Pegg, T.N. Bosman and P.J. Keuning  
Canada/netherlands experiment on pressure hull collapse - Model one: Out-of-circularity effects  
Technical memorandum 91/212, Defence research establishment atlantic, Dartmouth, Nova Scotia, Canada (August 1991)
  
- [5] W. Trouwborst  
Preliminary study on the design of a submarine's pressure hull  
TNO-report 5077006-88-1, 1988
  
- [6] W. Trouwborst  
Study on the interaction of buckling and collapse for imperfect infinitely long ring-stiffened cylinders  
TNO-report 5077014-89-1, 1989
  
- [7] W. Trouwborst  
Interaction of buckling and collapse and influence of residual stresses for a submarine's pressure hull  
TNO-report 5077025-90-1, 1990
  
- [8] W. Trouwborst  
Additional results for interaction and the influence of residual stresses for a submarine's pressure hull  
TNO-report B-92-0651, 1992



- [9] W. Trouwborst  
Buckling and collapse of the pressure hull of the Moray submarine class  
TNO-report B-91-0983, 1991
  
- [10] W. Trouwborst  
Elastic buckling for the modified Moray 1800 fore ship, annex to  
TNO-report B-91-0983
  
- [11] W. Trouwborst  
Global buckling and imperfection sensitivity of the pressure hull of the Moray submarine class  
TNO-report B-92-0187, 1992
  
- [12] W. Trouwborst  
Global collapse analysis using single frame model of the Moray submarine class including imperfections and residual stresses  
TNO-report B-92-0431, 1992
  
- [13] J.C. Wallace e.a.  
Finite element calculations for canada/netherlands pressure hull model tests  
Report DREA CR/90/451, MARTEC LTD, Halifax, Nova Scotia, Canada (November 1990)
  
- [14] J.C. Wallace and M.W. Chernuka  
Canada/netherlands model test analysis; Phase 3  
Report DREA CR/91/436, MARTEC LTD, Halifax, Nova Scotia, Canada (May 1991)
  
- [15] J.C. Wallace and M.W. Chernuka  
Canada/netherlands pressure hull model study; Update  
MARTEC LTD, Halifax, Nova Scotia, Canada (June 1991)

#### APPENDIX 1 MEASURED IMPERFECTIONS TEST CYL 1

For the first test cylinder (CYL 1) data concerning the imperfections is given in Table 5.1 of ref. [13], Table 2 of ref. [4], and in a computer output as delivered by the sponsor of this project. Radii for CYL 1 were measured every 15 degrees for ring number 4 to 8 (ring number 6 is central ring). Radii from the computer output are used as input for NAG-subroutines C06FPF, C06GSF, C06GQF and C06FQF, which perform the fourier analyses. These NAG-routines give results such that the fourier coefficients of  $n=12+i$  is the complex conjugate of  $n=12-i$ . Fourier coefficients ( $A_n$  corresponds with cosine terms and  $B_n$  with sine terms) correspond with the data from Table 5.1 of ref. [13] (up to the 4th decimal), but not with those from ref. [4]. This is caused by the fact that in ref. [4] the periodic signal which has been used for the fourier analyses includes both 0 and 360 degrees (if both 0 and 360 degrees are included, the NAG-routines give fourier coefficients corresponding with Table 2 of ref. [4]). The fourier coefficients as calculated by the NAG-routines have been used to calculate the radius as a function of the circumferential coordinate  $\theta$ . Error between these radii and the input radii is about  $1.e-8$ . Amplitudes for each  $n$  (from fourier coefficients), and amplitudes as a function of  $\theta$  have been calculated, both with respect to the original geometry with radius is 120.7 mm and with respect to the measured radius for  $n=0$ . Maximum and minimum amplitudes for central ring number 6 correspond with fig. 5.1 of ref. [13].

Results for ring number 4:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	121.377710	0	121.69235375	0.00000000	121.69235375	-0.31464374	0.67771001
15	121.438670	1	-0.02477005	-0.01238942	0.02769572	-0.25368375	0.73867000
30	121.621550	2	-0.03579880	-0.04559153	0.05796673	-0.07080376	0.92154999
45	121.776490	3	-0.21674396	-0.01960608	0.21762891	0.08413625	1.07649000
60	121.847610	4	-0.01185208	0.00733018	0.01393569	0.15525625	1.14761000
75	121.832370	5	-0.00246558	-0.00430216	0.00495859	0.14001625	1.13237000
90	121.735850	6	-0.02074333	-0.00317750	0.02098529	0.04349624	1.03584999
105	121.608850	7	-0.00066085	0.00234662	0.00243790	-0.08350376	0.90884999
120	121.530110	8	0.00126875	-0.00073540	0.00146647	-0.16224375	0.83011000
135	121.591070	9	-0.00211937	-0.00224941	0.00309057	-0.10128376	0.89106999
150	121.723150	10	-0.00060786	-0.00013097	0.00062181	0.03079624	1.02314999
165	121.832370	11	0.00037981	0.00085847	0.00093874	0.14001626	1.13237001
180	121.870470	12	-0.00053042	0.00000000		0.17811626	1.17047001
195	121.832370	13	0.00037981	-0.00085847		0.14001626	1.13237001
210	121.712990	14	-0.00060786	0.00013097		0.02063625	1.01299000
225	121.550430	15	-0.00211937	0.00224941		-0.14192375	0.85043000
240	121.448830	16	0.00126875	0.00073540		-0.24352375	0.74883000
255	121.537730	17	-0.00066085	-0.00234662		-0.15462374	0.83773001
270	121.740930	18	-0.02074333	0.00317750		0.04857624	1.04092999
285	121.888250	19	-0.00246558	0.00430216		0.19589626	1.18825001
300	121.951750	20	-0.01185208	-0.00733018		0.25939623	1.25174998
315	121.906030	21	-0.21674396	0.01960608		0.21367626	1.20603001
330	121.740930	22	-0.03579880	0.04559153		0.04857625	1.04093000
345	121.519980	23	-0.02477005	0.01238942		-0.17237374	0.81998001

Results for ring number 5:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	120.765570	0	121.62748500	0.00000000	121.62748500	-0.86191501	0.06556999
15	121.248170	1	-0.04460672	-0.01030924	0.04578252	-0.37931500	0.54817000
30	121.659650	2	-0.06125081	-0.05320547	0.08113251	0.03216501	0.95965001
45	121.832370	3	-0.39708943	-0.03262453	0.39842738	0.20488501	1.13237001
60	121.875550	4	-0.03904417	0.01410034	0.04151224	0.24806501	1.17555001
75	121.860310	5	-0.02157071	-0.01261659	0.02498947	0.23282497	1.16030997
90	121.763790	6	-0.15176500	-0.02347833	0.15357033	0.13630499	1.06378999
105	121.558050	7	-0.01470028	0.00332625	0.01507190	-0.06943501	0.85804999
120	121.138950	8	-0.00752250	-0.00423053	0.00863049	-0.48853501	0.43894999
135	121.481850	9	-0.07831391	-0.01315120	0.07941046	-0.14563499	0.78185001
150	121.748550	10	-0.00732919	0.00178714	0.00754393	0.12106500	1.04855000
165	121.857770	11	-0.00632896	-0.00169874	0.00655298	0.23028500	1.15777000
180	121.890790	12	-0.03239333	0.00000000		0.26330501	1.19079001
195	121.865390	13	-0.00632896	0.00169874		0.23790502	1.16539002
210	121.748550	14	-0.00732919	-0.00178714		0.12106500	1.04855000
225	121.494550	15	-0.07831391	0.01315120		-0.13293501	0.79454999
240	120.999250	16	-0.00752250	0.00423053		-0.62823499	0.29925001
255	121.405850	17	-0.01470028	-0.00332625		-0.22163500	0.70585000
270	121.773950	18	-0.15176500	0.02347833		0.14646501	1.07395001
285	121.939050	19	-0.02157071	0.01261659		0.31156499	1.23904999
300	121.989850	20	-0.03904417	-0.01410034		0.36236501	1.28985001
315	121.956830	21	-0.39708943	0.03262453		0.32934499	1.25682999
330	121.786650	22	-0.06125081	0.05320547		0.15916501	1.08665001
345	121.418350	23	-0.04460672	0.01030924		-0.20913501	0.71834999

Results for ring number 6 (central ring):

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as f(n) calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a f( $\theta$ ) [mm]	Amp. w.r.t. R=120.7 as f( $\theta$ ) [mm]
0	120.846850	0	121.62133958	0.00000000	121.62133958	-0.77448959	0.14684999
15	121.329450	1	-0.01581660	0.00014028	0.01581722	-0.29188959	0.62944999
30	121.672350	2	-0.03353912	-0.05953773	0.06833457	0.05101042	0.97235000
45	121.837450	3	-0.41716523	-0.02234798	0.41776340	0.21611041	1.13744999
60	121.878090	4	-0.01470958	0.02951054	0.03297338	0.25675039	1.17808997
75	121.862850	5	-0.00000693	-0.01691738	0.01691739	0.24151042	1.16285000
90	121.761250	6	-0.16785167	-0.01417917	0.16844949	0.13991043	1.06125001
105	121.532650	7	-0.00180391	0.01192898	0.01206460	-0.08868957	0.83265001
120	121.050050	8	0.00306792	-0.00385165	0.00492416	-0.57128958	0.35005000
135	121.481850	9	-0.08998810	-0.00668465	0.09023604	-0.13948958	0.78185000
150	121.748550	10	-0.00053922	0.00535356	0.00538065	0.12721041	1.04854999
165	121.862850	11	0.00154077	-0.00034275	0.00157843	0.24151043	1.16285001
180	121.893330	12	-0.03767792	0.00000000		0.27199041	1.19332999
195	121.875550	13	0.00154077	0.00034275		0.25421042	1.17555000
210	121.753630	14	-0.00053922	-0.00535356		0.13229040	1.05362998
225	121.436130	15	-0.08998810	0.00668465		-0.18520957	0.73613001
240	120.829070	16	0.00306792	0.00385165		-0.79226959	0.12906999
255	121.367580	17	-0.00180391	-0.01192898		-0.25375959	0.66757999
270	121.786650	18	-0.16785167	0.01417917		0.16531043	1.08665001
285	121.951750	19	-0.00000693	0.01691738		0.33041042	1.25175000
300	121.997470	20	-0.01470958	-0.02951054		0.37613042	1.29747000
315	121.951750	21	-0.41716523	0.02234798		0.33041042	1.25175000
330	121.786650	22	-0.03353912	0.05953773		0.16531041	1.08664999
345	121.418350	23	-0.01581660	-0.00014028		-0.20298959	0.71834999

Results for ring number 7:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Ampl. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	121.138950	0	121.63488500	0.00000000	121.63488500	-0.49593500	0.43895000
15	121.329450	1	-0.00847570	-0.00576694	0.01025159	-0.30543499	0.62945001
30	121.621550	2	-0.02638940	-0.06392522	0.06915804	-0.01333502	0.92154998
45	121.781570	3	-0.33757076	-0.02792389	0.33872373	0.14668500	1.08157000
60	121.850150	4	-0.00190500	0.02236366	0.02244465	0.21526500	1.15015000
75	121.837450	5	0.01799109	-0.01544034	0.02370829	0.20256500	1.13745000
90	121.743470	6	-0.10795000	-0.01989667	0.10976830	0.10858499	1.04346999
105	121.545350	7	0.00828895	0.01103618	0.01380232	-0.08953501	0.84534999
120	121.220230	8	0.01206500	-0.00696573	0.01393146	-0.41465500	0.52023000
135	121.489470	9	-0.05020257	-0.01099055	0.05139154	-0.14541500	0.78947000
150	121.735850	10	0.00733940	0.00719856	0.01028037	0.10096499	1.03584999
165	121.837450	11	0.01055899	-0.00642011	0.01235759	0.20256500	1.13745000
180	121.857770	12	-0.01968500	0.00000000		0.22288500	1.15777000
195	121.837450	13	0.01055899	0.00642011		0.20256499	1.13744999
210	121.723150	14	0.00733940	-0.00719856		0.08826501	1.02315001
225	121.481850	15	-0.05020257	0.01099055		-0.15303498	0.78185002
240	120.999250	16	0.01206500	0.00696573		-0.63563499	0.29925001
255	121.392950	17	0.00828895	-0.01103618		-0.24193500	0.69295000
270	121.761250	18	-0.10795000	0.01989667		0.12636501	1.06125001
285	121.931430	19	0.01799109	0.01544034		0.29654500	1.23143000
300	121.977150	20	-0.00190500	-0.02236366		0.34226499	1.27714999
315	121.921270	21	-0.33757076	0.02792389		0.28638498	1.22126998
330	121.753630	22	-0.02638940	0.06392522		0.11874502	1.05363002
345	121.469150	23	-0.00847570	0.00576694		-0.16573500	0.76915000

Results for ring number 8:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. $R n=0$ as a $f(\theta)$ [mm]	Amp. w.r.t. $R=120.7$ as $f(\theta)$ [mm]
0	121.405650	0	121.66282500	0.00000000	121.66282500	-0.25717501	0.70564999
15	121.436130	1	-0.01472550	-0.01330675	0.01984717	-0.22669499	0.73613001
30	121.583450	2	-0.03249938	-0.05542871	0.06425380	-0.07937500	0.88345000
45	121.728230	3	-0.20080615	-0.01039552	0.20107505	0.06540499	1.02822999
60	121.799350	4	-0.00031750	0.00604919	0.00605751	0.13652501	1.09935001
75	121.786650	5	0.00370442	-0.00373614	0.00526131	0.12382500	1.08665000
90	121.700290	6	-0.01608667	-0.00550333	0.01700199	0.03746501	1.00029001
105	121.588530	7	0.00217259	0.00175607	0.00279355	-0.07429499	0.88853001
120	121.507250	8	0.00285750	0.00018331	0.00286337	-0.15557500	0.80725000
135	121.570750	9	-0.00324052	-0.00277552	0.00426667	-0.09207498	0.87075002
150	121.705370	10	0.00159604	0.00103037	0.00189974	0.04254500	1.00537000
165	121.799350	11	0.00080516	0.00025104	0.00084339	0.13652499	1.09934999
180	121.829830	12	-0.00063500	0.00000000		0.16700499	1.12982999
195	121.773950	13	0.00080516	-0.00025104		0.11112500	1.07395000
210	121.652030	14	0.00159604	-0.00103037		-0.01079499	0.95203001
225	121.507250	15	-0.00324052	0.00277552		-0.15557501	0.80724999
240	121.413270	16	0.00285750	-0.00018331		-0.24955501	0.71326999
255	121.519950	17	0.00217259	-0.00175607		-0.14287501	0.81994999
270	121.723150	18	-0.01608667	0.00550333		0.06032501	1.02315001
285	121.870470	19	0.00370442	0.00373614		0.20764500	1.17047000
300	121.921270	20	-0.00031750	-0.00604919		0.25844500	1.22127000
315	121.860310	21	-0.20080615	0.01039552		0.19748500	1.16031000
330	121.705370	22	-0.03249938	0.05542871		0.04254499	1.00536999
345	121.519950	23	-0.01472550	0.01330675		-0.14287500	0.81995000

## APPENDIX 2 MEASURED IMPERFECTIONS TEST CYL 2

For the second test cylinder (CYL 2) data concerning the imperfections is given in Table 5.2 of ref. [13] and in a computer output as delivered by the sponsor of this project. These data is available as fourier coefficients for  $n=1$  to 11, and for all rings (1 to 11). Because fourier coefficients of the computer output corresponds with Table 5.2 of ref. [13], these coefficients are expected to be correct (see appendix 1, due to this correspondence these fourier coefficients are supposed to be calculated with only  $\theta=0$  degrees included in the periodic signal, not with both  $\theta=0$  and  $\theta=360$  degrees included). The fourier coefficients as given in the computer output have been used to calculate the amplitude for each  $n$  and the amplitude as a function of the circumferential coordinate  $\theta$ . Maximum and minimum amplitudes for central ring number 6 correspond exactly with fig. 5.2 of ref. [13].



Results for ring number 1:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as $f(n)$ delivered by sponsor	$B_n$ as $f(n)$ delivered by sponsor	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a $f(\theta)$ (mm)
1	-0.00117	0.00247	0.00273309	0	-0.11113000
2	-0.05120	0.06055	0.07929529	15	-0.10053988
3	-0.05521	-0.07371	0.09209402	30	-0.04760529
4	-0.00053	0.00275	0.00280061	45	0.05186036
5	0.00141	-0.00011	0.00141428	60	0.13016739
6	-0.00106	-0.00106	0.00149907	75	0.16616855
7	-0.00064	0.00066	0.00091935	90	0.13017000
8	0.00053	-0.00092	0.00106174	105	0.02645406
9	0.00017	0.00037	0.00040719	120	-0.07303808
10	-0.00171	0.00189	0.00254876	135	-0.15133746
11	-0.00172	-0.00042	0.00177054	150	-0.14922471
				165	-0.08783406
				180	0.00319000
				195	0.07725034
				210	0.10477402
				225	0.07725964
				240	0.01586808
				255	-0.02435901
				270	-0.02223000
				285	0.00105479
				300	0.02858261
				315	0.02645746
				330	-0.00952402
				345	-0.06243479

Results for ring number 2:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as f(n) delivered by sponsor	$B_n$ as f(n) delivered by sponsor	Ampl. as f(n) calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a f( $\theta$ ) [mm]
1	0.00874	0.00153	0.00887291	0	-0.22483000
2	-0.02258	0.07305	0.07646018	15	-0.15667180
3	-0.21066	-0.02336	0.21195123	30	0.04216359
4	0.00721	-0.01234	0.01429195	45	0.21134736
5	-0.00511	0.00500	0.00714927	60	0.28312217
6	-0.00492	-0.00642	0.00808844	75	0.22438932
7	-0.00925	-0.00596	0.01100382	90	0.08008000
8	0.00912	0.00873	0.01262487	105	-0.15654629
9	-0.00101	-0.00952	0.00957343	120	-0.28897468
10	-0.00425	0.00954	0.01044385	135	-0.25852390
11	0.00788	-0.00567	0.00970790	150	-0.08500359
				165	0.07251637
				180	0.19399000
				195	0.18651060
				210	-0.00197535
				225	-0.02950736
				240	-0.13596532
				255	-0.11854813
				270	0.01608000
				285	0.11038208
				300	0.14313783
				315	0.08432390
				330	-0.02182465
				345	-0.16967215

Results for ring number 3:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as $f(n)$ delivered by sponsor	$B_n$ as $f(n)$ delivered by sponsor	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a $f(\theta)$ (mm)
1	0.00605	-0.00793	0.00997434	0	-0.24871000
2	-0.05949	0.07519	0.09587803	15	-0.56725469
3	-0.25700	-0.34865	0.43313488	30	-0.29949436
4	0.01640	0.00458	0.01702752	45	0.10582767
5	-0.01075	0.01370	0.01741415	60	0.31009971
6	0.02222	-0.11430	0.11643976	75	0.44873918
7	0.00680	0.00686	0.00965917	90	0.36089000
8	-0.01111	0.00275	0.01144529	105	0.20742156
9	0.04533	-0.03750	0.05883076	120	-0.22332564
10	-0.00083	0.00736	0.00740665	135	-0.75775893
11	-0.00633	-0.00744	0.00976844	150	-0.42650564
				165	-0.02117321
				180	0.18309000
				195	0.35980852
				210	0.31009902
				225	0.23285233
				240	-0.09631436
				255	-0.47837301
				270	-0.27411000
				285	0.08043647
				300	0.20848029
				315	0.30903893
				330	0.19580098
				345	0.08043518

Results for ring number 4:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as f(n) delivered by sponsor	$B_n$ as f(n) delivered by sponsor	Ampl. as f(n) calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a f( $\theta$ ) (mm)
1	-0.00250	-0.01090	0.01118302	0	-0.30059000
2	-0.06369	0.06397	0.09026947	15	-0.78633792
3	-0.34075	-0.43470	0.55233563	30	-0.42757761
4	0.02540	-0.00917	0.02700461	45	0.12807559
5	-0.00999	-0.00275	0.01036159	60	0.38523605
6	0.02963	-0.14076	0.14384476	75	0.55985610
7	0.00787	0.00205	0.00813261	90	0.43605000
8	-0.00212	-0.00550	0.00589444	105	0.22966370
9	0.05288	-0.03676	0.06440180	120	-0.30056198
10	0.00230	0.00165	0.00283064	135	-0.86254621
11	0.00038	0.00123	0.00128736	150	-0.52916239
				165	0.02645690
				180	0.28363000
				195	0.45825070
				210	0.38523239
				225	0.22964441
				240	-0.17356802
				255	-0.60852889
				270	-0.32597000
				285	0.10264514
				300	0.28363395
				315	0.39474621
				330	0.28364761
				345	0.12807425

Results for ring number 5:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as f(n) delivered by sponsor	$B_n$ as f(n) delivered by sponsor	Ampl. as f(n) calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a f( $\theta$ ) [mm]
1	-0.00829	-0.01984	0.02150232	0	-0.30903000
2	-0.06996	0.07004	0.09899497	15	-0.99376282
3	-0.39722	-0.52704	0.65996582	30	-0.53762936
4	0.03281	-0.01833	0.03758304	45	0.14923199
5	-0.02610	-0.00936	0.02772760	60	0.45296332
6	0.05927	-0.17674	0.18641341	75	0.63182963
7	0.01030	-0.00090	0.01033925	90	0.50377000
8	0.00106	-0.01100	0.01105095	105	0.27621886
9	0.07125	-0.04021	0.08181324	120	-0.30904064
10	0.01069	0.00405	0.01143147	135	-1.04457241
11	0.00716	-0.01137	0.01343661	150	-0.66464064
				165	0.04763399
				180	0.37677000
				195	0.55563187
				210	0.42755146
				225	0.28892801
				240	-0.18202936
				255	-0.68897868
				270	-0.43603000
				285	0.12380999
				300	0.32598668
				315	0.47941241
				330	0.35135854
				345	0.17461716

Results for ring number 6 (central ring):

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as $f(n)$ delivered by sponsor	$B_n$ as $f(n)$ delivered by sponsor	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a $f(\theta)$ [mm]
1	0.00737	-0.02685	0.02784312	0	-0.31326000
2	-0.05896	0.08429	0.10286440	15	-0.89217468
3	-0.40506	-0.54062	0.67553208	30	-0.56726802
4	0.05609	-0.02200	0.06025021	45	0.12382181
5	-0.03139	-0.00141	0.03142165	60	0.47413524
6	0.05927	-0.16192	0.17242685	75	0.65723105
7	0.01559	0.01129	0.01924869	90	0.52492000
8	-0.01376	-0.00733	0.01559059	105	0.25081318
9	0.06640	-0.03262	0.07397989	120	-0.33866524
10	-0.00030	0.01096	0.01096411	135	-1.14616078
11	-0.00851	-0.01415	0.01651189	150	-0.69426198
				165	0.04760233
				180	0.39794000
				195	0.58103153
				210	0.44874012
				225	0.25081819
				240	-0.18624476
				255	-0.66356790
				270	-0.44028000
				285	0.09842787
				300	0.32171476
				315	0.49212078
				330	0.37252988
				345	0.20003662

Results for ring number 7:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as $f(n)$ delivered by sponsor	$B_n$ as $f(n)$ delivered by sponsor	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a $f(\theta)$ [mm]
1	-0.03235	0.02439	0.04051413	0	-0.32634000
2	-0.02482	0.02037	0.03210871	15	-0.74392042
3	-0.35711	-0.43970	0.56644827	30	-0.54132818
4	0.00567	-0.03407	0.03453858	45	0.06908719
5	0.03285	-0.00269	0.03295995	60	0.42367834
6	0.01483	-0.10058	0.10166742	75	0.57710175
7	0.00329	-0.01638	0.01670714	90	0.47368000
8	0.01425	0.03017	0.03336602	105	0.17010199
9	0.02461	-0.02287	0.03359597	120	-0.38934605
10	0.00073	-0.00345	0.00352639	135	-0.62991628
11	-0.00829	0.01041	0.01330760	150	-0.33833182
				165	0.01808715
				180	0.34766000
				195	0.50009152
				210	0.39764868
				225	0.18307281
				240	-0.23732395
				255	-0.71891286
				270	-0.41532000
				285	0.04305691
				300	0.27065166
				315	0.41207628
				330	0.33467132
				345	0.12007395

Results for ring number 8:

n	Input given by Fourier components			Amplitude of imperf.	
	$A_n$ as $f(n)$ delivered by sponsor	$B_n$ as $f(n)$ delivered by sponsor	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a $f(\theta)$ (mm)
1	0.00409	-0.02083	0.02122774	0	-0.29527000
2	-0.03830	0.06845	0.07843655	15	-0.70168227
3	-0.33644	-0.39769	0.52091190	30	-0.35877395
4	0.04286	-0.02475	0.04949285	45	0.11113816
5	-0.02046	0.00588	0.02128817	60	0.35242268
6	0.00317	-0.12594	0.12597989	75	0.51752985
7	0.01101	0.00335	0.01150837	90	0.41594000
8	-0.00476	-0.00275	0.00549728	105	0.17461137
9	0.04223	-0.03785	0.05670975	120	-0.38417047
10	0.00020	0.00246	0.00246812	135	-0.86677346
11	0.00113	-0.00137	0.00177589	150	-0.46037605
				165	0.07301117
				180	0.30161000
				195	0.45401692
				210	0.35242686
				225	0.18732184
				240	-0.19367953
				255	-0.53656450
				270	-0.26988000
				285	0.07303119
				300	0.23810732
				315	0.37783346
				330	0.30164314
				345	0.13652628



Results for ring number 9:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as $f(n)$ delivered by sponsor	$B_n$ as $f(n)$ delivered by sponsor	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a $f(\theta)$ [mm]
1	-0.00062	-0.01482	0.01483296	0	-0.20953000
2	-0.03353	0.05462	0.06409060	15	-0.58418924
3	-0.25189	-0.31120	0.40036735	30	-0.26034395
4	0.02540	-0.01466	0.02932705	45	0.08889402
5	-0.01469	-0.00108	0.01472965	60	0.26032210
6	0.00635	-0.09948	0.09968246	75	0.40639847
7	0.00630	-0.00235	0.00672402	90	0.31114000
8	-0.00317	-0.00550	0.00634814	105	0.13969034
9	0.04446	-0.03604	0.05723262	120	-0.26035198
10	0.00496	0.00147	0.00517325	135	-0.66039763
11	0.00690	-0.00508	0.00856834	150	-0.36194605
				165	0.03809384
				180	0.20955000
				195	0.36830446
				210	0.27303686
				225	0.16510598
				240	-0.13335802
				255	-0.41911369
				270	-0.22224000
				285	0.06349815
				300	0.17146790
				315	0.29211763
				330	0.22225314
				345	0.10159767

Results for ring number 10:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as f(n) delivered by sponsor	$B_n$ as f(n) delivered by sponsor	Ampl. as f(n) calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a f( $\theta$ ) [mm]
1	0.00242	-0.01051	0.01078501	0	-0.12384000
2	-0.01421	0.04768	0.04975245	15	-0.20278285
3	-0.12800	-0.15585	0.20167603	30	-0.13652488
4	0.00836	-0.00642	0.01054068	45	0.02582072
5	-0.00031	0.00004	0.00031257	60	0.16828146
6	0.00106	-0.00889	0.00895297	75	0.22903225
7	0.00059	0.00166	0.00176173	90	0.16828000
8	0.00116	-0.00092	0.00148054	105	0.00042942
9	0.00311	0.00079	0.00320877	120	-0.17463471
10	0.00045	0.00185	0.00190394	135	-0.27896826
11	0.00153	-0.00155	0.00217793	150	-0.21273512
				165	-0.03765553
				180	0.11748000
				195	0.20362774
				210	0.18097849
				225	0.07661928
				240	-0.07301529
				255	-0.15197713
				270	-0.12384000
				285	-0.01230081
				300	0.09208854
				315	0.14772826
				330	0.11748151
				345	0.00042692

Results for ring number 11:

Input given by Fourier components				Amplitude of imperf.	
n	$A_n$ as $f(n)$ delivered by sponsor	$B_n$ as $f(n)$ delivered by sponsor	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	$\theta$ (deg.)	Ampl. as a $f(\theta)$ [mm]
1	0.00114	-0.01413	0.01417591	0	-0.05968000
2	-0.00302	0.04612	0.04621877	15	-0.07258667
3	-0.06062	-0.06532	0.08911502	30	-0.03935294
4	0.00011	-0.00385	0.00385157	45	0.03407743
5	0.00122	0.00047	0.00130740	60	0.09270043
6	0.00042	0.00106	0.00114018	75	0.10520084
7	-0.00053	-0.00036	0.00064070	90	0.05967000
8	0.00116	-0.00018	0.00117388	105	-0.04213887
9	0.00135	0.00156	0.00206303	120	-0.11305522
10	-0.00120	0.00066	0.00136953	135	-0.1437159
11	0.00029	-0.00102	0.00106042	150	-0.11301706
				165	-0.02941563
				180	0.05462000
				195	0.11030417
				210	0.10540554
				225	0.05946257
				240	-0.00381478
				255	-0.04721835
				270	-0.04953000
				285	-0.01163896
				300	0.03174957
				315	0.05438859
				330	0.03430466
				345	-0.01670654

### APPENDIX 3 MEASURED IMPERFECTIONS TEST CYL 5

For the third test cylinder (CYL 5) data concerning the imperfections is given in appendix C of ref. [15]. Radii for CYL 5 were measured every 10 degrees for ring number 1 to 11 (ring number 6 is central ring). These radii are used as input for NAG-subroutines C06FPF, C06GSF, C06GQF and C06FQF, which perform the fourier analyses. Fourier coefficients for  $n=18+i$  form the complex conjugate of coefficients for  $n=18-i$ . The fourier coefficients as calculated by the NAG-routines have been used to calculate the radius as a function of the circumferential coordinate  $\theta$ . Errors between these radii and the input radii are about  $1.e-8$ . Amplitudes for each  $n$  (from fourier coefficients), and amplitudes as a function of  $\theta$  have been calculated, both with respect to the original geometry with radius is 120.7 mm and with respect to the measured radius for  $n=0$ . Maxima and minima correspond with fig. 1.4 c) of ref. [15].

Results for ring number 1:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	$n$	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. $R n=0$ as a $f(\theta)$ [mm]	Amp. w.r.t. $R=120.7$ as $f(\theta)$ [mm]
0	120.487400	0	120.48280833	0.00000000	120.48280833	0.00459167	-0.21260000
10	120.487400	1	0.01704453	0.01475753	0.02254553	0.00459167	-0.21260000
20	120.507800	2	0.12406782	0.01915151	0.12553726	0.02499167	-0.19220000
30	120.533200	3	-0.13499367	0.00457946	0.13507132	0.05039167	-0.16680000
40	120.550900	4	0.00919705	0.00264994	0.00957120	0.06809167	-0.14910000
50	120.545900	5	-0.00082669	-0.00072377	0.00109875	0.06309167	-0.15410000
60	120.520500	6	-0.00953333	0.00280977	0.00993878	0.03769167	-0.17950000
70	120.490000	7	0.00137600	-0.00093223	0.00166206	0.00719167	-0.21000000
80	120.423900	8	-0.00136606	0.00019862	0.00138042	-0.05890833	-0.27610000
90	120.368100	9	-0.00029444	0.00085556	0.00090481	-0.11470833	-0.33190000
100	120.304600	10	0.00050144	-0.00064317	0.00081554	-0.17820833	-0.39540000
110	120.266500	11	-0.00063346	0.00019121	0.00066169	-0.21630833	-0.43350000
120	120.266500	12	0.00021667	-0.00158771	0.00160243	-0.21630833	-0.43350000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
130	120.317300	13	-0.00083706	-0.00052312	0.00098708	-0.16550833	-0.38270000
140	120.418900	14	0.00093908	0.00036928	0.00100907	-0.06390833	-0.28110000
150	120.545900	15	0.00065478	0.00262609	0.00270649	0.06309167	-0.15410000
160	120.647500	16	-0.00085599	-0.00061823	0.00105590	0.16469167	-0.05250000
170	120.705900	17	0.00036000	-0.00155165	0.00159286	0.22309167	0.00590000
180	120.723700	18	-0.00042500	0.00000000		0.24089167	0.02370000
190	120.698300	19	0.00036000	0.00155165		0.21549167	-0.00170000
200	120.622100	20	-0.00085599	0.00061823		0.13929167	-0.07790000
210	120.533200	21	0.00065478	-0.00262609		0.05039167	-0.16680000
220	120.413800	22	0.00093908	-0.00036928		-0.06900833	-0.28620000
230	120.317300	23	-0.00083706	0.00052312		-0.16550833	-0.38270000
240	120.261400	24	0.00021667	0.00158771		-0.22140833	-0.43860000
250	120.261400	25	-0.00063346	-0.00019121		-0.22140833	-0.43860000
260	120.317300	26	0.00050144	0.00064317		-0.16550833	-0.38270000
270	120.380800	27	-0.00029444	-0.00085556		-0.10200833	-0.31920000
280	120.469700	28	-0.00136606	-0.00019862		-0.01310833	-0.23030000
290	120.525500	29	0.00137600	0.00093223		0.04269167	-0.17450000
300	120.578900	30	-0.00953333	-0.00280977		0.09609167	-0.12110000
310	120.609400	31	-0.00082669	0.00072377		0.12659167	-0.09060000
320	120.609400	32	0.00919705	-0.00264994		0.12659167	-0.09060000
330	120.596700	33	-0.13499367	-0.00457946		0.11389167	-0.10330000
340	120.558600	34	0.12406782	-0.01915151		0.07579167	-0.14140000
350	120.515400	35	0.01704453	-0.01475753		0.03259167	-0.18460000

Results for ring number 2:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. $R$ $n=0$ as a $f(\theta)$ [mm]	Amp. w.r.t. $R=120.7$ as $f(\theta)$ [mm]
0	120.152200	0	120.45847222	0.00000000	120.45847222	-0.30627222	-0.54780000
10	120.317300	1	0.01848095	0.01701108	0.02511817	-0.14117222	-0.38270000
20	120.490000	2	0.15769314	0.00609717	0.15781097	0.03152778	-0.21000000
30	120.596700	3	-0.34574751	0.00854683	0.34585313	0.13822778	-0.10330000
40	120.642400	4	0.03231169	0.00113030	0.03233146	0.18392778	-0.05760000
50	120.647500	5	0.00387079	-0.00261287	0.00467013	0.18902778	-0.05250000
60	120.629700	6	-0.10569444	0.00366617	0.10575801	0.17122778	-0.07030000
70	120.584000	7	0.01123397	0.00375196	0.01184396	0.12552778	-0.11600000
80	120.507800	8	0.00415392	-0.00468829	0.00626379	0.04932778	-0.19220000
90	120.406200	9	-0.05221667	0.00380556	0.05235516	-0.05227222	-0.29380000
100	120.266500	10	0.00887422	0.00157782	0.00901339	-0.19197222	-0.43350000
110	120.050600	11	0.00453279	-0.00242362	0.00514005	-0.40787222	-0.64940000
120	119.728000	12	-0.03132222	-0.00048113	0.03132592	-0.73047222	-0.97200000
130	120.063300	13	0.00661886	0.00199390	0.00691266	-0.39517222	-0.63670000
140	120.368100	14	0.00382431	-0.00268627	0.00467347	-0.09037222	-0.33190000
150	120.596700	15	-0.02240249	-0.00050794	0.02240825	0.13822778	-0.10330000
160	120.754100	16	0.00480939	0.00334684	0.00585931	0.29562778	0.05410000
170	120.850700	17	0.00352931	-0.00448044	0.00570354	0.39222778	0.15070000
180	120.896400	18	-0.00882222	0.00000000		0.43792778	0.19640000
190	120.868400	19	0.00352931	0.00448044		0.40992778	0.16840000
200	120.769400	20	0.00480939	-0.00334684		0.31092778	0.06940000
210	120.596700	21	-0.02240249	0.00050794		0.13822778	-0.10330000
220	120.393500	22	0.00382431	0.00268627		-0.06497222	-0.30650000
230	120.088700	23	0.00661886	-0.00199390		-0.36977222	-0.61130000
240	119.796600	24	-0.03132222	0.00048113		-0.66187222	-0.90340000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
250	120.037900	25	0.00453279	0.00242362		-0.42057222	-0.66210000
260	120.266500	26	0.00887422	-0.00157782		-0.19197222	-0.43350000
270	120.418900	27	-0.05221667	-0.00380556		-0.03957222	-0.28110000
280	120.540800	28	0.00415392	0.00468829		0.08232778	-0.15920000
290	120.617000	29	0.01123397	-0.00375196		0.15852778	-0.08300000
300	120.672900	30	-0.10569444	-0.00366617		0.21442778	-0.02710000
310	120.693200	31	0.00387079	0.00261287		0.23472778	-0.00680000
320	120.685600	32	0.03231169	-0.00113030		0.22712778	-0.01440000
330	120.634800	33	-0.34574751	-0.00854683		0.17632778	-0.06520000
340	120.533200	34	0.15769314	-0.00609717		0.07472778	-0.16680000
350	120.342700	35	0.01848095	-0.01701108		-0.11577222	-0.35730000

Results for ring number 3:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	120.126800	0	120.47251667	0.00000000	120.47251667	-0.34571667	-0.57320000
10	120.266500	1	0.01377846	0.00759085	0.01573109	-0.20601667	-0.43350000
20	120.469700	2	0.17847210	0.00389877	0.17851468	-0.00281667	-0.23030000
30	120.609400	3	-0.43412377	0.01214480	0.43429361	0.13688333	-0.09060000
40	120.705900	4	0.04412550	-0.01058495	0.04537732	0.23338333	0.00590000
50	120.741400	5	0.00092688	0.00995396	0.00999702	0.26888333	0.04140000
60	120.731300	6	-0.09673889	0.00671651	0.09697177	0.25878333	0.03130000
70	120.660200	7	0.01671002	-0.00357058	0.01708724	0.18768333	-0.03980000
80	120.558600	8	0.00017068	0.00220303	0.00220964	0.08608333	-0.14140000
90	120.393500	9	-0.04346667	0.00677778	0.04399192	-0.07901667	-0.30650000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with MAG- subroutine	$B_n$ calculat. with MAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
100	120.215700	10	0.01181855	-0.00210475	0.01200451	-0.25681667	-0.48430000
110	120.012500	11	-0.00190285	0.00167050	0.00253208	-0.46001667	-0.68750000
120	119.733100	12	-0.02505000	0.00354108	0.02529905	-0.73941667	-0.96690000
130	119.961700	13	0.00739106	0.00062153	0.00741714	-0.51081667	-0.73830000
140	120.317300	14	0.00127601	-0.00048983	0.00136680	-0.15521667	-0.38270000
150	120.596700	15	-0.01814290	0.00309965	0.01840578	0.12418333	-0.10330000
160	120.825300	16	0.00333715	-0.00005806	0.00333766	0.35278333	0.12530000
170	120.965000	17	0.00162978	0.00110024	0.00196639	0.49248333	0.26500000
180	121.041200	18	-0.00592778	0.00000000		0.56868333	0.34120000
190	121.003100	19	0.00162978	-0.00110024		0.53058333	0.30310000
200	120.863400	20	0.00333715	0.00005806		0.39088333	0.16340000
210	120.647500	21	-0.01814290	-0.00309965		0.17498333	-0.05250000
220	120.355400	22	0.00127601	0.00048983		-0.11711667	-0.34460000
230	119.987100	23	0.00739106	-0.00062153		-0.48541667	-0.71290000
240	119.687300	24	-0.02505000	-0.00354108		-0.78521667	-1.01270000
250	119.949000	25	-0.00190285	-0.00167050		-0.52351667	-0.75100000
260	120.215700	26	0.01181855	0.00210475		-0.25681667	-0.48430000
270	120.418900	27	-0.04346667	-0.00677778		-0.05361667	-0.28110000
280	120.584000	28	0.00017068	-0.00220303		0.11148333	-0.11600000
290	120.685600	29	0.01671002	0.00357058		0.21308333	-0.01440000
300	120.749100	30	-0.09673889	-0.00671651		0.27658333	0.04910000
310	120.761800	31	0.00092688	-0.00995396		0.28928333	0.06180000
320	120.723700	32	0.04412550	0.01058495		0.25118333	0.02370000
330	120.634800	33	-0.43412377	-0.01214480		0.16228333	-0.06520000
340	120.495100	34	0.17847210	-0.00389877		0.02258333	-0.20490000
350	120.317300	35	0.01377846	-0.00759085		-0.15521667	-0.38270000



Results for ring number 4:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with MAG- subroutine	$B_n$ calculat. with MAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	120.063300	0	120.47787500	0.00000000	120.47787500	-0.41457500	-0.63670000
10	120.215700	1	0.00559946	-0.00015157	0.00560152	-0.26217500	-0.48430000
20	120.444300	2	0.19513750	-0.00347168	0.19516838	-0.03357500	-0.25570000
30	120.634800	3	-0.51098459	0.01032268	0.51108885	0.15692500	-0.06520000
40	120.766800	4	0.04394742	-0.01800824	0.04749392	0.28892500	0.06680000
50	120.825300	5	0.00264168	0.01966594	0.01984257	0.34742500	0.12530000
60	120.807500	6	-0.09878056	0.00439267	0.09887818	0.32962500	0.10750000
70	120.723700	7	0.01439518	-0.00347692	0.01480912	0.24582500	0.02370000
80	120.584000	8	0.00748478	0.00810540	0.01103265	0.10612500	-0.11600000
90	120.393500	9	-0.04530000	0.00352778	0.04543716	-0.08437500	-0.30650000
100	120.177600	10	0.00915233	-0.00207636	0.00938490	-0.30027500	-0.52240000
110	119.910900	11	0.00273217	0.00656337	0.00710933	-0.56697500	-0.78910000
120	119.682300	12	-0.02765833	0.00049556	0.02766277	-0.79557500	-1.01770000
130	119.936300	13	0.00655194	-0.00246621	0.00700072	-0.54157500	-0.76370000
140	120.291900	14	0.00230184	0.00651931	0.00691375	-0.18597500	-0.40810000
150	120.596700	15	-0.02069874	-0.00044490	0.02070352	0.11882500	-0.10330000
160	120.868400	16	0.00487613	-0.00467357	0.00675417	0.39052500	0.16840000
170	121.053900	17	0.00151290	0.00508829	0.00530844	0.57602500	0.35390000
180	121.150400	18	-0.00748611	0.00000000		0.67252500	0.45040000
190	121.104700	19	0.00151290	-0.00508829		0.62682500	0.40470000
200	120.947200	20	0.00487613	0.00467357		0.46932500	0.24720000
210	120.672900	21	-0.02069874	0.00044490		0.19502500	-0.62710000
220	120.342700	22	0.00230184	-0.00651931		-0.13517500	-0.35730000
230	119.936300	23	0.00655194	0.00246621		-0.54157500	-0.76370000
240	119.555300	24	-0.02765833	-0.00049556		-0.92257500	-1.14470000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
250	119.885500	25	0.00273217	-0.00656337		-0.59237500	-0.81450000
260	120.164900	26	0.00915233	0.00207636		-0.31297500	-0.53510000
270	120.418900	27	-0.04530000	-0.00352778		-0.05897500	-0.28110000
280	120.609400	28	0.00748478	-0.00810540		0.13152500	-0.09060000
290	120.731300	29	0.01439518	0.00347692		0.25342500	0.03130000
300	120.804900	30	-0.09878056	-0.00439267		0.32702500	0.10490000
310	120.807500	31	0.00264168	-0.01966594		0.32962500	0.10750000
320	120.749100	32	0.04394742	0.01800824		0.27122500	0.04910000
330	120.622100	33	-0.51098459	-0.01032268		0.14422500	-0.07790000
340	120.457000	34	0.19513750	0.00347168		-0.02087500	-0.24300000
350	120.266500	35	0.00559946	0.00015157		-0.21137500	-0.43350000

Results for ring number 5:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	119.949000	0	120.47540833	0.00000000	120.47540833	-0.52640833	-0.75100000
10	120.190300	1	0.00742849	-0.01439920	0.01620246	-0.28510833	-0.50970000
20	120.444300	2	0.18967100	-0.00711598	0.18980444	-0.03110833	-0.25570000
30	120.647500	3	-0.58517647	0.00500531	0.58519787	0.17209167	-0.05250000
40	120.812600	4	0.05215323	-0.02901089	0.05967907	0.33719167	0.11260000
50	120.888800	5	-0.00096230	0.02364282	0.02366239	0.41339167	0.18880000
60	120.888800	6	-0.10993611	0.00000481	0.10993611	0.41339167	0.18880000
70	120.799900	7	0.01605885	-0.01224287	0.02019343	0.32449167	0.09990000
80	120.622100	8	-0.00046125	0.00853585	0.00854831	0.14669167	-0.07790000
90	120.418900	9	-0.05022778	-0.00212222	0.05027259	-0.05650833	-0.28110000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
100	120.164900	10	0.00680309	-0.00583628	0.00896349	-0.31050833	-0.53510000
110	119.860100	11	-0.00272761	0.00612750	0.00670717	-0.61530833	-0.83990000
120	119.644200	12	-0.02455833	-0.00366136	0.02482977	-0.83120833	-1.05580000
130	119.860100	13	-0.00088347	-0.00433884	0.00442787	-0.61530833	-0.83990000
140	120.241100	14	-0.00153242	0.00383448	0.00412935	-0.23430833	-0.45890000
150	120.596700	15	-0.01692909	0.00133914	0.01698197	0.12129167	-0.10330000
160	120.901500	16	0.00143302	-0.00533595	0.00552502	0.42609167	0.20150000
170	121.104700	17	-0.00028062	0.00379653	0.00380689	0.62929167	0.40470000
180	121.216400	18	-0.00628056	0.00000000		0.74099167	0.51640000
190	121.163100	19	-0.00028062	-0.00379653		0.68769167	0.46310000
200	120.977700	20	0.00143302	0.00533595		0.50229167	0.27770000
210	120.698300	21	-0.01692909	-0.00133914		0.22289167	-0.00170000
220	120.291900	22	-0.00153242	-0.00383448		-0.18350833	-0.40810000
230	119.809300	23	-0.00088347	0.00433884		-0.66610833	-0.89070000
240	119.453700	24	-0.02455833	0.00366136		-1.02170833	-1.24630000
250	119.796600	25	-0.00272761	-0.00612750		-0.67880833	-0.90340000
260	120.152200	26	0.00680309	0.00583628		-0.32320833	-0.54780000
270	120.431600	27	-0.05022778	0.00212222		-0.04380833	-0.26840000
280	120.647500	28	-0.00046125	-0.00853585		0.17209167	-0.05250000
290	120.787200	29	0.01605885	0.01224287		0.31179167	0.08720000
300	120.855700	30	-0.10993611	-0.00000481		0.38029167	0.15570000
310	120.843000	31	-0.00096230	-0.02364282		0.36759167	0.14300000
320	120.761800	32	0.05215323	0.02901089		0.28639167	0.06180000
330	120.609400	33	-0.58517647	-0.00500531		0.13399167	-0.09060000
340	120.418900	34	0.18967100	0.00711598		-0.05650833	-0.28110000
350	120.164900	35	0.00742849	0.01439920		-0.31050833	-0.53510000

Results for ring number 6 (central ring):

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. $R=0$ as a $f(\theta)$ [mm]	Ampl. w.r.t. $R=120.7$ as $f(\theta)$ [mm]
0	119.974400	0	120.48936944	0.00000000	120.48936944	-0.51496944	-0.72560000
10	120.215700	1	0.01042966	-0.01678100	0.01975803	-0.27366944	-0.48430000
20	120.449300	2	0.18371701	-0.01399221	0.18424907	-0.04006944	-0.25070000
30	120.672900	3	-0.59405149	0.0069039	0.59409158	0.18353056	-0.02710000
40	120.779500	4	0.06004035	-0.02285857	0.06424451	0.29013056	0.07950000
50	120.926900	5	0.00352386	0.01833043	0.01866607	0.43753056	0.22690000
60	120.931900	6	-0.10216667	-0.00586973	0.10233514	0.44253056	0.23190000
70	120.850700	7	0.01475237	-0.01350304	0.01999911	0.36133056	0.15070000
80	120.672900	8	-0.00157651	0.00605429	0.00625618	0.18353056	-0.02710000
90	120.444300	9	-0.05024444	-0.00380556	0.05038836	-0.04506944	-0.25570000
100	120.177600	10	0.00479581	-0.00099231	0.00489740	-0.31176944	-0.52240000
110	119.860100	11	-0.00284688	0.00404348	0.00494515	-0.62926944	-0.83990000
120	119.631500	12	-0.02201111	-0.00098150	0.02203298	-0.85786944	-1.06850000
130	119.860100	13	0.00434247	-0.00349682	0.00557537	-0.62926944	-0.83990000
140	120.241100	14	0.00115385	0.00055801	0.00128169	-0.24826944	-0.45890000
150	120.596700	15	-0.01187073	-0.00435694	0.01264504	0.10733056	-0.10330000
160	120.901500	16	-0.00214717	-0.00630551	0.00666107	0.41213056	0.20150000
170	121.142800	17	-0.00268481	-0.00116217	0.00292555	0.65343056	0.44280000
180	121.231700	18	-0.00812500	0.00000000		0.74233056	0.53170000
190	121.180900	19	-0.00268481	0.00116217		0.69153056	0.48090000
200	120.990400	20	-0.00214717	0.00630551		0.50103056	0.29040000
210	120.698300	21	-0.01187073	0.00435694		0.20893056	-0.00170000
220	120.291900	22	0.00115385	-0.00055801		-0.19746944	-0.40810000
230	119.809300	23	0.00434247	0.00349682		-0.68006944	-0.89070000
240	119.496800	24	-0.02201111	0.00098150		-0.99256944	-1.20320000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. $R n=0$ as a $f(\theta)$ [mm]	Amp. w.r.t. $R=120.7$ as $f(\theta)$ [mm]
250	119.809300	25	-0.00284688	-0.00404348		-0.68006944	-0.89070000
260	120.164900	26	0.00479581	0.00099231		-0.32446944	-0.53510000
270	120.444300	27	-0.05024444	0.00380556		-0.04506944	-0.25570000
280	120.665200	28	-0.00157651	-0.00605429		0.17583056	-0.03480000
290	120.804900	29	0.01475237	0.01350304		0.31553056	0.10490000
300	120.876100	30	-0.10216667	0.00586973		0.38673056	0.17610000
310	120.863400	31	0.00352386	-0.01833043		0.37403056	0.16340000
320	120.766800	32	0.06004035	0.02285857		0.27743056	0.06680000
330	120.609400	33	-0.59405149	-0.00690139		0.12003056	-0.09060000
340	120.418900	34	0.18371701	0.01399221		-0.07046944	-0.28110000
350	120.164900	35	0.01042966	0.01678100		-0.32446944	-0.53510000

Results for ring number 7:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. $R n=0$ as a $f(\theta)$ [mm]	Amp. w.r.t. $R=120.7$ as $f(\theta)$ [mm]
0	119.961700	0	120.49523333	0.00000000	120.49523333	-0.53353333	-0.73830000
10	120.215700	1	0.02824615	-0.02130214	0.03537833	-0.27953333	-0.48430000
20	120.469700	2	0.16485641	-0.03117222	0.16777766	-0.02553333	-0.23030000
30	120.685600	3	-0.58346854	0.00930784	0.58354278	0.19036667	-0.01440000
40	120.850700	4	0.05665991	-0.00690336	0.05707891	0.35546667	0.15070000
50	120.939600	5	0.00298891	0.00334238	0.00448387	0.44436667	0.23960000
60	120.952300	6	-0.10838611	0.00244893	0.10841377	0.45706667	0.25230000
70	120.883700	7	0.01321186	-0.00178269	0.01333159	0.38846667	0.18370000
80	120.723700	8	0.00487773	0.00193022	0.00524576	0.22846667	0.02370000
90	120.495100	9	-0.05263333	0.00493333	0.05286403	-0.00013333	-0.20490000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
100	120.215700	10	0.00699655	0.00036676	0.00700616	-0.27953333	-0.48430000
110	119.885500	11	0.00101923	-0.00003327	0.00101977	-0.60973333	-0.81450000
120	119.555300	12	-0.03359167	0.00489304	0.03394616	-0.93993333	-1.14470000
130	119.834700	13	0.00191672	-0.00147745	0.00242005	-0.66053333	-0.86530000
140	120.253800	14	-0.00314463	0.00037650	0.00316708	-0.24143333	-0.44620000
150	120.584000	15	-0.02133146	0.00197549	0.02142273	0.08876667	-0.11600000
160	120.888800	16	-0.00077931	-0.00033185	0.00084703	0.39356667	0.18880000
170	121.104700	17	-0.00334954	-0.00142875	0.00364153	0.60946667	0.40470000
180	121.188500	18	-0.00762222	0.00000000		0.69326667	0.48850000
190	121.130100	19	-0.00334954	0.00142875		0.63486667	0.43010000
200	120.952300	20	-0.00077931	0.00033185		0.45706667	0.25230000
210	120.647500	21	-0.02133146	-0.00197549		0.15226667	-0.05250000
220	120.304600	22	-0.00314463	-0.00037650		-0.19063333	-0.39540000
230	119.885500	23	0.00191672	0.00147745		-0.60973333	-0.81450000
240	119.547600	24	-0.03359167	-0.00489304		-0.94763333	-1.15240000
250	119.834700	25	0.00101923	0.00003327		-0.66053333	-0.86530000
260	120.190300	26	0.00699655	-0.00036676		-0.30493333	-0.50970000
270	120.444300	27	-0.05263333	-0.00493333		-0.05093333	-0.25570000
280	120.660200	28	0.00487773	-0.00193022		0.16496667	-0.03980000
290	120.799900	29	0.01321186	0.00178269		0.30466667	0.09990000
300	120.868400	30	-0.10838611	-0.00244893		0.37316667	0.16840000
310	120.855700	31	0.00298891	-0.00334238		0.36046667	0.15570000
320	120.774500	32	0.05665991	0.00690336		0.27926667	0.07450000
330	120.609400	33	-0.58346854	-0.00930784		0.11416667	-0.09060000
340	120.418900	34	0.16485641	0.03117222		-0.07633333	-0.28110000
350	120.215700	35	0.02824615	0.02130214		-0.27953333	-0.48430000

Results for ring number 8:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	119.910900	0	120.50023611	0.00000000	120.50023611	-0.58933611	-0.78910000
10	120.190300	1	0.02690565	-0.01314192	0.02994368	-0.30993611	-0.50970000
20	120.469700	2	0.13163044	-0.03979110	0.13751329	-0.03053611	-0.23030000
30	120.711000	3	-0.54541777	0.00503292	0.54544099	0.21076389	0.01100000
40	120.850700	4	0.03820505	0.00509463	0.03854323	0.35046389	0.15070000
50	120.931900	5	0.00035973	0.00363648	0.00365423	0.43166389	0.23190000
60	120.944600	6	-0.12370278	0.00329571	0.12374667	0.44436389	0.24460000
70	120.876100	7	0.01271914	0.00531054	0.01378327	0.37586389	0.17610000
80	120.723700	8	-0.00280205	0.00108753	0.00300570	0.22346389	0.02370000
90	120.520500	9	-0.05925556	0.00607222	0.05956587	0.02026389	-0.17950000
100	120.241100	10	0.00923141	0.00193552	0.00943213	-0.25913611	-0.45890000
110	119.936300	11	-0.00244428	0.00293822	0.00382199	-0.56393611	-0.76370000
120	119.555300	12	-0.03563611	0.00207365	0.03569639	-0.94493611	-1.14470000
130	119.885500	13	0.00403764	-0.00183554	0.00443529	-0.61473611	-0.81450000
140	120.279200	14	-0.00116185	0.00139870	0.00181831	-0.22103611	-0.42080000
150	120.622100	15	-0.03071001	-0.00107737	0.03072890	0.12186389	-0.07790000
160	120.850700	16	0.00269701	-0.00034093	0.00271847	0.35046389	0.15070000
170	121.020800	17	0.00075545	-0.00157692	0.00174854	0.52056389	0.32080000
180	121.097000	18	-0.01474722	0.00000000		0.59676389	0.39700000
190	121.053900	19	0.00075545	0.00157692		0.55366389	0.35390000
200	120.909100	20	0.00269701	0.00034093		0.40886389	0.20910000
210	120.660200	21	-0.03071001	0.00107737		0.15996389	-0.03980000
220	120.342700	22	-0.00116185	-0.00139870		-0.15753611	-0.35730000
230	119.961700	23	0.00403764	0.00183554		-0.53853611	-0.73830000
240	119.606100	24	-0.03563611	-0.00207365		-0.89413611	-1.09390000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
250	119.961700	25	-0.00244428	-0.00293822		-0.53853611	-0.73830000
260	120.241100	26	0.00923141	-0.00193552		-0.25913611	-0.45890000
270	120.482400	27	-0.05925556	-0.00607222		-0.01783611	-0.21760000
280	120.667800	28	-0.00280205	-0.00108753		0.16756389	-0.03220000
290	120.787200	29	0.01271914	-0.00531054		0.28696389	0.08720000
300	120.843000	30	-0.12370278	-0.00329571		0.34276389	0.14300000
310	120.830300	31	0.00035973	-0.00363648		0.33006389	0.13030000
320	120.761800	32	0.03820505	-0.00509463		0.26156389	0.06180000
330	120.622100	33	-0.54541777	-0.00503292		0.12186389	-0.07790000
340	120.444300	34	0.13163044	0.03979110		-0.05593611	-0.25570000
350	120.215700	35	0.02690565	0.01314192		-0.28453611	-0.48430000

Results for ring number 9:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	120.042900	0	120.51166667	0.00000000	120.51166667	-0.46876667	-0.65710000
10	120.241100	1	0.02752342	-0.00456415	0.02789928	-0.27056667	-0.45890000
20	120.482400	2	0.09794872	-0.04506952	0.10782028	-0.02926667	-0.21760000
30	120.685600	3	-0.44560988	0.00473949	0.44563509	0.17393333	-0.01440000
40	120.812600	4	0.02809693	0.01429207	0.03152301	0.30093333	0.11260000
50	120.881100	5	0.00049921	0.00004365	0.00050112	0.36943333	0.18110000
60	120.888800	6	-0.10393611	0.00280496	0.10397395	0.37713333	0.18880000
70	120.838000	7	0.01016922	0.00457782	0.01115210	0.32633333	0.13800000
80	120.711000	8	-0.00035934	0.00086438	0.00093610	0.19933333	0.01100000
90	120.533200	9	-0.04671667	0.00198333	0.04675875	0.02153333	-0.16680000



Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
100	120.304600	10	0.00763991	0.00237390	0.00800023	-0.20706667	-0.39540000
110	120.012500	11	0.00027947	-0.00018455	0.00033490	-0.49916667	-0.68750000
120	119.720400	12	-0.02589167	-0.00012990	0.02589199	-0.79126667	-0.97960000
130	119.987100	13	0.00314588	-0.00128735	0.00339910	-0.52456667	-0.71290000
140	120.317300	14	0.00235304	0.00124097	0.00266022	-0.19436667	-0.38270000
150	120.596700	15	-0.01864012	-0.00063949	0.01865108	0.08503333	-0.10330000
160	120.787200	16	0.00188742	-0.00022080	0.00190029	0.27553333	0.08720000
170	120.926900	17	0.00194947	-0.00038222	0.00198659	0.41523333	0.22690000
180	120.977700	18	-0.00910556	0.00000000		0.46603333	0.27770000
190	120.939600	19	0.00194947	0.00038222		0.42793333	0.23960000
200	120.830300	20	0.00188742	0.00022080		0.31863333	0.13030000
210	120.647500	21	-0.01864012	0.00063949		0.13583333	-0.05250000
220	120.393500	22	0.00235304	-0.00124097		-0.11816667	-0.30650000
230	120.088700	23	0.00314588	0.00128735		-0.42296667	-0.61130000
240	119.822000	24	-0.02589167	0.00012990		-0.68966667	-0.87800000
250	120.088700	25	0.00027947	0.00018455		-0.42296667	-0.61130000
260	120.324900	26	0.00763991	-0.00237390		-0.18676667	-0.37510000
270	120.507800	27	-0.04671667	-0.00198333		-0.00386667	-0.19220000
280	120.652500	28	-0.00035934	-0.00086438		0.14083333	-0.04750000
290	120.741400	29	0.01016922	-0.00457782		0.22973333	0.04140000
300	120.784600	30	-0.10393611	-0.00280496		0.27293333	0.08460000
310	120.782100	31	0.00049921	-0.00004365		0.27043333	0.08210000
320	120.723700	32	0.02809693	-0.01429207		0.21203333	0.02370000
330	120.622100	33	-0.44560988	-0.00473949		0.11043333	-0.07790000
340	120.469700	34	0.09794872	0.04506952		-0.04196667	-0.23030000
350	120.253800	35	0.02752342	0.00456415		-0.25786667	-0.44620000

Results for ring number 10:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. $R_{n=0}$ as a $f(\theta)$ [mm]	Amp. w.r.t. $R=120.7$ as $f(\theta)$ [mm]
0	120.126800	0	120.52619167	0.00000000	120.52619167	-0.39939167	-0.57320000
10	120.304600	1	0.02721253	0.01785099	0.03254504	-0.22159167	-0.39540000
20	120.500100	2	0.04322824	-0.03829623	0.05775190	-0.02609167	-0.19990000
30	120.647500	3	-0.32996740	-0.00208727	0.32997401	0.12130833	-0.05250000
40	120.741400	4	0.03161255	0.01223289	0.03389686	0.21520833	0.04140000
50	120.789700	5	0.00225510	0.00695717	0.00731353	0.26350833	0.08970000
60	120.799900	6	-0.09892500	0.00391155	0.09900230	0.27370833	0.09990000
70	120.766800	7	0.00951163	-0.00268056	0.00988213	0.24060833	0.06680000
80	120.685600	8	0.00896371	0.00244792	0.00929195	0.15940833	-0.01440000
90	120.571300	9	-0.04473889	0.01128889	0.04614117	0.04510833	-0.12870000
100	120.380800	10	-0.00093462	0.00174728	0.00198154	-0.14539167	-0.31920000
110	120.164900	11	-0.00163600	-0.00548840	0.00572704	-0.36129167	-0.53510000
120	119.877800	12	-0.01777500	0.00121725	0.01781663	-0.64839167	-0.82220000
130	120.114100	13	0.00360279	0.00894854	0.00964658	-0.41209167	-0.58590000
140	120.368100	14	-0.00989362	0.00006200	0.00989381	-0.15809167	-0.33190000
150	120.571300	15	-0.01984371	-0.00820718	0.02147395	0.04510833	-0.12870000
160	120.698300	16	0.01085708	0.00306107	0.01128035	0.17210833	-0.00170000
170	120.792200	17	0.00055396	0.00504101	0.00507136	0.26600833	0.09220000
180	120.832900	18	-0.01347500	0.00000000		0.30670833	0.13290000
190	120.812600	19	0.00055396	-0.00504101		0.28640833	0.11260000
200	120.741400	20	0.01085708	-0.00306107		0.21520833	0.04140000
210	120.622100	21	-0.01984371	0.00820718		0.09590833	-0.07790000
220	120.469700	22	-0.00989362	-0.00006200		-0.05649167	-0.23030000
230	120.215700	23	0.00360279	-0.00894854		-0.31049167	-0.48430000
240	119.999800	24	-0.01777500	-0.00121725		-0.52639167	-0.70020000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
250	120.241100	25	-0.00163600	0.00548840		-0.28509167	-0.45890000
260	120.431600	26	-0.00093462	-0.00174728		-0.09459167	-0.26840000
270	120.708400	27	-0.04473889	-0.01128889		0.18220833	0.00840000
280	120.655100	28	0.00896371	-0.00244792		0.12890833	-0.04490000
290	120.716000	29	0.00951163	0.00268056		0.18980833	0.01600000
300	120.738900	30	-0.09892500	-0.00391155		0.21270833	0.03890000
310	120.728700	31	0.00225510	-0.00695717		0.20250833	0.02870000
320	120.685600	32	0.03161255	-0.01223289		0.15940833	-0.01440000
330	120.604300	33	-0.32996740	0.00208727		0.07810833	-0.09570000
340	120.495100	34	0.04322824	0.03829623		-0.03109167	-0.20490000
350	120.342700	35	0.02721253	-0.01785099		-0.18349167	-0.35730000

Results for ring number 11:

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with NAG- subroutine	$B_n$ calculat. with NAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
0	120.444300	0	120.56455556	0.00000000	120.56455556	-0.12025556	-0.25570000
10	120.469700	1	0.01703476	0.00925058	0.01938443	-0.09485556	-0.23030000
20	120.533200	2	-0.00537255	-0.02980809	0.03028839	-0.03135556	-0.16680000
30	120.596700	3	-0.13043835	0.00003444	0.13043835	0.03214444	-0.10330000
40	120.657600	4	0.00990559	0.00575025	0.01145365	0.09304444	-0.04240000
50	120.698300	5	0.00011612	-0.00179199	0.00179575	0.13374444	-0.00170000
60	120.716000	6	-0.00959722	0.00097668	0.00964679	0.15144444	0.01600000
70	120.698300	7	-0.00010197	0.00038770	0.00040088	0.13374444	-0.00170000
80	120.652500	8	-0.00010761	-0.00042741	0.00044075	0.08794444	-0.04750000
90	120.584000	9	-0.00128333	-0.00028333	0.00131424	0.01944444	-0.11600000

Input		Results Fourier analyses				Amplitudes of imperfection	
$\theta$ (deg)	Measured radius [mm]	n	$A_n$ calculat. with MAG- subroutine	$B_n$ calculat. with MAG- subroutine	Ampl. as $f(n)$ calc. from $A_n$ and $B_n$	Ampl. w.r.t. R n=0 as a $f(\theta)$ [mm]	Amp. w.r.t. R=120.7 as $f(\theta)$ [mm]
100	120.495100	10	-0.00061932	0.00077500	0.00099206	-0.06945556	-0.20490000
110	120.411200	11	-0.00007629	0.00068181	0.00068607	-0.15335556	-0.28880000
120	120.373100	12	-0.00028056	-0.00000481	0.00028060	-0.19145556	-0.32690000
130	120.385800	13	0.00016028	-0.00019953	0.00025593	-0.17875556	-0.31420000
140	120.449300	14	-0.00035813	0.00050314	0.00061758	-0.11525556	-0.25070000
150	120.525500	15	0.00007168	0.00051556	0.00052052	-0.03905556	-0.17450000
160	120.601700	16	0.00040202	-0.00031756	0.00051231	0.03714444	-0.09830000
170	120.652500	17	0.00021710	0.00099378	0.00101722	0.08794444	-0.04750000
180	120.672900	18	0.00007222	0.00000000		0.10834444	-0.02710000
190	120.665200	19	0.00021710	-0.00099378		0.10064444	-0.03480000
200	120.629700	20	0.00040202	0.00031756		0.06514444	-0.07030000
210	120.576300	21	0.00007168	-0.00051556		0.01174444	-0.12370000
220	120.515400	22	-0.00035813	-0.00050314		-0.04915556	-0.18460000
230	120.469700	23	0.00016028	0.00019953		-0.09485556	-0.23030000
240	120.451900	24	-0.00028056	0.00000481		-0.11265556	-0.24810000
250	120.482400	25	-0.00007629	-0.00068181		-0.08215556	-0.21760000
260	120.533200	26	-0.00061932	-0.00077500		-0.03135556	-0.16680000
270	120.596700	27	-0.00128333	0.00028333		0.03214444	-0.10330000
280	120.644900	28	-0.00010761	0.00042741		0.08034444	-0.05510000
290	120.670300	29	-0.00010197	-0.00038770		0.10574444	-0.02970000
300	120.670300	30	-0.00959722	-0.00097668		0.10574444	-0.02970000
310	120.652500	31	0.00011612	0.00179199		0.08794444	-0.04750000
320	120.614400	32	0.00990559	-0.00575025		0.04984444	-0.08560000
330	120.563600	33	-0.13043835	-0.00003444		-0.00095556	-0.13640000
340	120.507800	34	-0.00537255	0.02980809		-0.05675556	-0.19220000
350	120.462000	35	0.01703476	-0.00925058		-0.10255556	-0.23800000

APPENDIX 4 RESULTS OF STAGE 2.2, ANALYTICAL CALCULATIONS

Sheet	Method	Remarks	$P_{subbul}$	$\delta_{mat}$	$\delta_{weld}$	$\delta_{shape}$	$\delta_{res}$	$\delta_{int}$	$\delta_{nonaxi}$	$\gamma_{analnum}$	$\gamma_{num}$	$P_{real}$
Axisymmetrical critical and collapse pressures, infinitely long												
1	Lumpy	Midbay	7.02	0.96	1.0	1.0	1.0	1.0	1.0	1.00	0.95	6.40
1	Pulos	Midbay	7.00	0.96	1.0	1.0	1.0	1.0	1.0	1.00	0.95	6.38
1	Pulos	At stiff	6.27	0.96	1.0	1.0	1.0	1.0	1.0	1.00	0.95	5.72
2	Pulos	End web	11.47	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	9.42
3	Lunpc	Midbay	7.67	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	6.30
3	-	At stiff	-	-	-	-	-	-	-	-	-	5.72
4	-	End web	-	-	-	-	-	-	-	-	-	9.42
Elastic global buckling pressures, infinitely long												
5	Kengl1	n=2	3.49	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	2.75
5	Kengl1	n=3	9.89	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	7.80
5	Kengl1	n=4	19.90	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	15.7
5	Kengl1	n=5	25.70	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	20.3
Elastic local buckling pressures												
6	Kenl11	n=2	51.68	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	43.9
6	Kenl11	n=3	44.86	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	38.1
6	Kenl11	n=4	38.18	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	32.4
6	Kenl11	n=5	32.73	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	27.8
6	Kenl11	n=6	28.89	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	24.6
6	Kenl11	n=7	26.61	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	22.6
6	Kenl11	n=8	25.64	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	21.8
6	Kenl11	n=9	25.71	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	21.8
6	Kenl11	n=10	26.58	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	22.6
6	Kenl11	n=11	28.07	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	23.9
6	Kenl11	n=12	30.06	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	25.5
6	Kenl11	n=13	32.46	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	27.6
6	Kenl11	n=14	35.21	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	29.9

Sheet	Method	Remarks	$P_{subhul}$	$S_{mat}$	$S_{weld}$	$S_{shape}$	$S_{res}$	$S_{int}$	$S_{nonaxi}$	$\gamma_{analnum}$	$\gamma_{num}$	$P_{real}$
6	Ken111	n=15	38.28	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	32.5
6	Ken111	n=16	41.63	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	35.4
6	Ken111	n=17	45.26	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	38.5
6	Ken111	n=18	49.14	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	41.8
6	Ken111	n=19	53.27	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	45.3
6	Ken111	n=20	57.64	0.96	1.0	0.98	0.98	1.0	1.0	0.97	0.95	49.0
Elastic tilt buckling pressures												
7	Pkn0	n=0	58.19	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.95	48.4
7	Pkploi	n=0	56008	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.95	4.E4
Critical pressures for the hull midbay in case of a local imperfection (amplitude is 0.1 mm)												
8	Knikv.	n=2	6.92	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.68
8	Knikv.	n=3	6.86	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.63
8	Knikv.	n=4	6.78	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.57
8	Knikv.	n=5	6.68	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.48
8	Knikv.	n=6	6.56	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.38
8	Knikv.	n=7	6.45	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.29
8	Knikv.	n=8	6.36	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.22
8	Knikv.	n=9	6.30	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.17
8	Knikv.	n=10	6.26	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.14
8	Knikv.	n=11	6.23	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.11
8	Knikv.	n=12	6.22	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.11
8	Knikv.	n=13	6.21	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.10
8	Knikv.	n=14	6.21	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.10
8	Knikv.	n=15-25	6.20	0.96	1.0	1.0	1.0	0.9	1.0	1.00	0.95	5.09
Elastic global buckling pressures, finite compartment												
10	BS5500	n=2	11.43	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	9.01
10	BS5500	n=3	11.55	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	9.10
10	BS5500	n=4	20.01	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	15.8
10	BS5500	n=5	30.78	0.96	1.0	0.98	0.98	1.0	1.0	0.95	0.90	24.3

Sheet	Method	Remarks	$P_{subbul}$	$S_{mat}$	$S_{weld}$	$S_{shape}$	$S_{res}$	$S_{int}$	$S_{nonaxi}$	$\gamma_{analnum}$	$\gamma_{num}$	$P_{real}$
Axisymmetrical critical pressures, finite (until disturbances from end cap have disappeared)												
12	PREBUCK	t=12 cap	12.87	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	10.6
12	PREBUCK	At ring 1	7.52	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	6.17
12	PREBUCK	Mid R 1/2	7.00	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	5.74
12	PREBUCK	At ring 2	6.38	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	5.24
12	PREBUCK	Mid R 2/3	7.19	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	5.90
12	PREBUCK	At ring 3	6.43	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	5.28
12	PREBUCK	end web 1	11.62	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	9.54
12	PREBUCK	end web 2	11.49	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	9.43
12	PREBUCK	end web 3	11.55	0.96	1.0	1.0	1.0	1.0	1.0	0.90	0.95	9.48

Table A4.1 Results of stage 2.2, independent of global imperfection, in MPa

Critical pressures for the web, global imperfection, infinitely long												
Fixed data:												
Sheet		Method		$\beta_{mat}$	$\beta_{weld}$	$\beta_{shape}$	$\beta_{res}$	$\beta_{int}$	$\beta_{nonaxi}$		$\gamma_{num}$	
9		BS5500		0.96	1.0	1.0	1.0	0.9	1.0		0.95	
	Results for CYL 1				Results for CYL 2				Results for CYL 5			
n	W0	$P_{subhul}$	$\gamma_{analnum}$	$P_{real}$	W0	$P_{subhul}$	$\gamma_{analnum}$	$P_{real}$	W0	$P_{subhul}$	$\gamma_{analnum}$	$P_{real}$
2	0.08	3.95	0.90	2.9	0.10	3.90	0.90	2.9	0.20	3.60	0.90	2.7
3	0.42	5.37	0.98	4.3	0.68	4.46	0.98	3.6	0.60	4.68	0.98	3.8
4	0.042	9.99	0.98	8.0	0.06	9.65	0.98	7.8	0.065	9.54	0.98	7.7
5	0.025	10.6	0.98	8.5	0.033	10.4	0.98	8.4	0.025	10.6	0.98	8.5
3	0.86	4.03	0.98	3.2	1.15	3.49	0.98	2.8	1.02	3.70	0.98	3.0
Critical pressures for the web, global imperfection, finite cylinders												
Fixed data:												
Sheet		Method		$\beta_{mat}$	$\beta_{weld}$	$\beta_{shape}$	$\beta_{res}$	$\beta_{int}$	$\beta_{nonaxi}$		$\gamma_{num}$	
11		BS5500		0.96	1.0	1.0	1.0	0.9	1.0		0.95	
	Results for CYL 1				Results for CYL 2				Results for CYL 5			
n	W0	$P_{subhul}$	$\gamma_{analnum}$	$P_{real}$	W0	$P_{subhul}$	$\gamma_{analnum}$	$P_{real}$	W0	$P_{subhul}$	$\gamma_{analnum}$	$P_{real}$
2	0.08	9.29	0.90	6.9	0.10	9.08	0.94	7.0	0.20	8.31	0.94	6.4
3	0.42	5.58	0.98	4.5	0.68	4.65	0.98	3.7	0.60	4.90	0.98	3.9
4	0.042	10.0	0.98	8.0	0.060	9.66	0.98	7.8	0.065	9.56	0.98	7.7
5	0.025	10.6	0.98	8.5	0.033	10.4	0.98	8.4	0.025	10.6	0.98	8.5
3	0.86	4.19	0.98	3.4	1.15	3.64	0.98	2.9	1.02	3.87	0.98	3.1

Table A4.2 Results in case of a global imperfection as calculated according to the design methodology, stage 2.2, pressures in MPa and W0 in mm



# APPENDIX 5 RESULTS OF STAGE 3, NUMERICAL CALCULATIONS, INFINITELY LONG

Calculations are carried out according to appendix III of the design methodology. Up to  $n=14$ , the smallest buckling pressure is a global one. Therefore local buckling pressures for wave number smaller than 14 are calculated including the suppression of the radial displacements at the connection between web and hull during buckling. Starting from  $n=14$ , this extra boundary condition does not influence the buckling mode anymore. Buckling modes become as given in fig. 3.3 c).

Sheet	Method	Remarks	$P_{b0smar}$	$\beta_{mat}$	$\beta_{weld}$	$\beta_{shape}$	$\beta_{res}$	$\beta_{int}$	$\beta_{nonaxi}$	$\gamma_{num}$	$P_{real}$
Axisymmetrical critical and collapse pressures, infinitely long											
1	BOSOR5	Midbay	7.1	0.96	1.0	1.0	1.0	1.0	1.0	0.95	6.48
1	BOSOR5	At stiff	6.3	0.96	1.0	1.0	1.0	1.0	1.0	0.95	5.75
2	BOSOR5	Begin web	12.0	0.96	1.0	1.0	1.0	1.0	1.0	0.95	10.9
3	BOSOR5		9.05	0.96	1.0	1.0	1.0	1.0	1.0	0.95	8.25
4	BOSOR5		> 41.	0.96	1.0	1.0	1.0	1.0	1.0	0.95	-
Elastic global buckling pressures, infinitely long											
5	BOSOR5	$n=2$	3.8	0.96	1.0	0.98	0.98	1.0	1.0	0.90	3.15
5	BOSOR5	$n=3$	9.5	0.96	1.0	0.98	0.98	1.0	1.0	0.90	7.88
5	BOSOR5	$n=4$	16.2	0.96	1.0	0.98	0.98	1.0	1.0	0.90	13.4
5	BOSOR5	$n=5$	22.5	0.96	1.0	0.98	0.98	1.0	1.0	0.90	18.7
Elastic tilt and local buckling pressures											
5	BOSOR5	$n=0$	63.3	0.96	1.0	0.98	0.98	1.0	1.0	0.95	55.4
5	BOSOR5	$n=1$	60.1	0.96	1.0	0.98	0.98	1.0	1.0	0.95	52.6
5	BOSOR5	$n=2$	53.6	0.96	1.0	0.98	0.98	1.0	1.0	0.95	46.9
5	BOSOR5	$n=3$	47.3	0.96	1.0	0.98	0.98	1.0	1.0	0.95	41.4
5	BOSOR5	$n=4$	42.3	0.96	1.0	0.98	0.98	1.0	1.0	0.95	37.0
5	BOSOR5	$n=5$	38.8	0.96	1.0	0.98	0.98	1.0	1.0	0.95	34.0
5	BOSOR5	$n=6$	36.7	0.96	1.0	0.98	0.98	1.0	1.0	0.95	32.1
5	BOSOR5	$n=7$	35.8	0.96	1.0	0.98	0.98	1.0	1.0	0.95	31.4
5	BOSOR5	$n=8$	35.7	0.96	1.0	0.98	0.98	1.0	1.0	0.95	31.3

Sheet	Method	Remarks	$P_{\text{bosmar}}$	$\beta_{\text{mat}}$	$\beta_{\text{weld}}$	$\beta_{\text{shape}}$	$\beta_{\text{res}}$	$\beta_{\text{int}}$	$\beta_{\text{nonaxi}}$	$\gamma_{\text{num}}$	$P_{\text{real}}$
5	BOSOR5	n=9	36.3	0.96	1.0	0.98	0.98	1.0	1.0	0.95	31.8
5	BOSOR5	n=10	37.4	0.96	1.0	0.98	0.98	1.0	1.0	0.95	32.8
5	BOSOR5	n=11	38.9	0.96	1.0	0.98	0.98	1.0	1.0	0.95	34.1
5	BOSOR5	n=12	40.7	0.96	1.0	0.98	0.98	1.0	1.0	0.95	35.6
5	BOSOR5	n=13	42.8	0.96	1.0	0.98	0.98	1.0	1.0	0.95	37.5
5	BOSOR5	n=14	44.9	0.96	1.0	0.98	0.98	1.0	1.0	0.95	39.3
5	BOSOR5	n=15	44.6	0.96	1.0	0.98	0.98	1.0	1.0	0.95	39.1
5	BOSOR5	n=16	47.0	0.96	1.0	0.98	0.98	1.0	1.0	0.95	41.2
5	BOSOR5	n=17	49.8	0.96	1.0	0.98	0.98	1.0	1.0	0.95	43.6
5	BOSOR5	n=18	52.8	0.96	1.0	0.98	0.98	1.0	1.0	0.95	46.2
5	BOSOR5	n=19	56.1	0.96	1.0	0.98	0.98	1.0	1.0	0.95	49.1
5	BOSOR5	n=20	59.9	0.96	1.0	0.98	0.98	1.0	1.0	0.95	52.5
Plastic global buckling pressures, infinitely long											
6	BOSOR5	n=3	8.44	0.96	1.0	1.0	0.95	1.0	1.0	0.90	6.93
6	BOSOR5	n=4	8.99	0.96	1.0	1.0	0.95	1.0	1.0	0.90	7.38
6	BOSOR5	n=5	> $P_{\text{el}}$	0.96	1.0	1.0	0.95	1.0	1.0	0.90	-
Plastic tilt and local buckling pressures											
6	BOSOR5	n=0	8.82	0.96	1.0	1.0	1.0	1.0	1.0	0.90	7.62
6	BOSOR5	n=1	8.84	0.96	1.0	1.0	1.0	1.0	1.0	0.90	7.64
6	BOSOR5	n=6-10	> $P_{\text{el}}$	0.96	1.0	1.0	1.0	1.0	1.0	0.90	-
Critical/collapse pressures in case of n=0 tilt imperfection ( $W_0=0.1$ mm)											
7	BOSOR5	crit-midbay	7.0	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.75
7	BOSOR5	crit-at stf	6.3	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.17
7	BOSOR5	coll-hull	9.0	0.96	1.0	1.0	0.95	0.9	1.0	0.95	7.02
7	BOSOR5	crit-web	11.9	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.77

Sheet	Method	Remarks	$P_{b0smar}$	$\beta_{mat}$	$\beta_{weld}$	$\beta_{shape}$	$\beta_{res}$	$\beta_{int}$	$\beta_{nonaxi}$	$\gamma_{num}$	$P_{real}$
Critical pressures for the hull midbay in case of a tilt or local imperfection (amplitude is 0.1 mm), as calculated with BOSPOST											
7,9	Midway	n=0	6.72	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.52
	At stif	n=0	6.20	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.09
	End web	n=0	11.58	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.50
7,9	Midway	n=1	6.71	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.51
	At stif	n=1	6.20	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.09
	End web	n=1	11.58	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.50
7,9	Midway	n=2	6.67	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.47
	At stif	n=2	6.20	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.09
	End web	n=2	11.56	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.49
7,9	Midway	n=3	6.62	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.43
	At stif	n=3	6.20	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.09
	End web	n=3	11.55	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.48
7,9	Midway	n=4	6.56	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.38
	At stif	n=4	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=4	11.51	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.45
7,9	Midway	n=5	6.50	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.34
	At stif	n=5	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=5	11.48	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.42
7,9	Midway	n=6	6.44	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.29
	At stif	n=6	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=6	11.45	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.40
7,9	Midway	n=7	6.39	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.24
	At stif	n=7	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=7	11.43	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.38
7,9	Midway	n=8	6.36	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.22
	At stif	n=8	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=8	11.40	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.36

Sheet	Method	Remarks	$P_{\text{bosmar}}$	$\beta_{\text{mat}}$	$\beta_{\text{weld}}$	$\beta_{\text{shape}}$	$\beta_{\text{res}}$	$\beta_{\text{int}}$	$\beta_{\text{nonaxi}}$	$\gamma_{\text{num}}$	$P_{\text{real}}$
7,9	Midway	n=9	6.31	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.18
	At stif	n=9	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=9	11.39	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.35
7,9	Midway	n=10	6.28	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.15
	At stif	n=10	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=10	11.40	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.36
7,9	Midway	n=11	6.25	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.13
	At stif	n=11	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=11	11.39	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.35
7,9	Midway	n=12	6.23	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.11
	At stif	n=12	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=12	11.38	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.34
7,9	Midway	n=13	6.21	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.10
	At stif	n=13	6.19	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.08
	End web	n=13	11.38	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.34
7,9	Midway	n=14	6.20	0.96	1.0	1.0	1.0	0.9	1.0	0.95	5.09
	At stif	n=14	5.69	0.96	1.0	1.0	1.0	0.9	1.0	0.95	4.67
	End web	n=14	11.65	0.96	1.0	1.0	1.0	0.9	1.0	0.95	9.56

Table A5.1 Results as calculated for stage 3 with BOSOR5 and BOSPOST, independent of global imperfection, infinitely long, pressures in MPa

n	W0 (mm)	P <sub>calc</sub>	P <sub>real</sub>	P <sub>calc</sub>	P <sub>real</sub>	P <sub>calc</sub>	P <sub>real</sub>
Model CYL 1		Hull midbay		Hull at web		Web	
2	0.08	3.71	3.04	3.68	3.02	3.62	2.98
3	0.42	5.71	4.64	5.11	4.20	5.39	4.43
4	0.042	6.85	5.64	6.11	5.01	11.09	9.10
5	0.025	6.90	5.66	6.14	5.04	11.43	9.38
3	0.86	5.03	4.13	4.29	3.52	4.07	3.33
Model CYI 2		Hull midbay		Hull at web		Web	
2	0.10	3.71	3.04	3.65	2.93	3.58	2.94
3	0.68	5.28	4.43	4.59	3.76	4.51	3.71
4	0.060	6.79	5.57	6.07	4.98	10.61	8.71
5	0.033	6.87	5.64	6.11	5.01	11.36	9.32
3	1.15	4.70	3.86	3.89	3.19	3.54	2.90
Model CYL 5		Hull midbay		Hull at web		Web	
2	0.20	3.64	2.99	3.52	2.88	3.39	2.73
3	0.60	5.40	4.44	4.74	3.89	4.74	3.89
4	0.065	6.77	5.51	6.06	4.97	10.48	8.60
5	0.025	6.90	5.66	6.14	5.04	11.43	9.37
3	1.02	4.84	3.97	4.06	3.33	3.76	3.08
Only factors different from 1.0 are: $\beta_{mat} = 0.96$ , $\beta_{int} = 0.9$ and $\gamma_{num} = 0.95$							

Table A5.2 Critical pressures in MPa as calculated by BOSPOST according to NUM-CALC-8 (stage 3) in case of global imperfections, infinitely long

Marc model	P <sub>start</sub> [MPa]	n	P <sub>calc</sub> (P <sub>k</sub> ) [MPa]	P <sub>k</sub> live load [MPa]	P <sub>k</sub> MARC/P <sub>k</sub> BOSOR5
n0 (8 deg)	58.0	0	59.988	59.988	0.95
n1 (180 deg)	3.0	1	66.73	66.73	1.11
n2	3.0	2 g	5.539	4.154	1.09
n3	8.0	3 g	11.359	10.097	1.06
		3 l	51.46	51.46	1.09
n4	15.0	4 g	17.995	16.870	1.04
		4 l	44.12	44.12	1.04
n5	20.0	5 g	24.02	23.059	1.02
		5 l	39.38	39.38	1.01
n6	25.0	6 g	28.58	27.786	1.01
		6 l	35.84	35.84	0.98
n7	28.0	7 g	31.539	30.895	1.00
		7 l	34.219	34.219	0.96
n8	30.0	8 g	33.42	32.898	0.99
		8 l	33.62	33.62	0.94
		16 g	48.305	48.116	1.02
		16 l	47.56	47.56	1.02
n9	30.0	9 g	34.67	34.242	0.99
		9 l	33.685	33.685	0.93
		18 g	54.29	54.122	1.03
		18 l	53.77	53.77	1.03
n10	33.0	10 g	35.875	35.516	0.99
		10 l	34.45	34.45	0.92
		20 g	60.11	59.96	1.00
		20 l	59.87	59.87	1.00

Marc model	P <sub>start</sub> [MPa]	n	P <sub>calc</sub> (P <sub>k</sub> ) [MPa]	P <sub>k</sub> live load [MPa]	P <sub>k</sub> MARC/P <sub>k</sub> BOSOR5
n11	35.0	11 g	37.23	36.922	0.99
		11 l	35.745	35.745	0.92
		22 g	67.665	67.525	
		22 l	67.535	67.535	
n12	35.0	12 g	38.545	38.277	0.99
		12 l	37.135	37.135	0.91
		24 g	75.73	75.599	
		24 l	75.64	75.64	
n13	38.0	13 g	40.102	39.865	0.99
		13 l	38.806	38.806	0.91
		26 g	81.66	81.539	
		26 l	81.62	81.62	
n14	40.0	14 g	42.075	41.860	0.99
		14 l	40.965	40.965	0.91
		28 g	90.4	90.285	
		28 l	90.35	90.35	
n15	40.0	15 g	43.98	43.785	0.98
		15 l	43.055	43.055	
		30 g	99.3	99.19	
		30 l	99.3	99.3	
P <sub>start</sub> is pressure load from which eigenvalue analysis is started g denotes global buckling mode, l denotes local buckling mode					

Table A5.3 Comparison of elastic buckling pressures for infinitely long cylinders, BOSOR5 and MARC results

n	W0 [mm]	P <sub>calc</sub> crit hul Midbay	P <sub>calc</sub> crit hul At web	P <sub>calc</sub> critical web	P <sub>calc</sub> collapse	P <sub>real</sub> crit hul Midbay	P <sub>real</sub> crit hul At web	P <sub>real</sub> critic. web	P <sub>real</sub> col- lapse
Axisymmetrical perfect case									
0	-	7.0	6.4	> P <sub>coll</sub>	8.9	6.38	5.84	-	8.11
0	Hull el.	-	-	11.9	> 20.0	-	-	10.85	-
Local/tilt imperfections									
0	0.1	6.7	6.4	> P <sub>coll</sub>	8.7	5.44	5.25	-	6.78
8	0.1	6.4	6.2	> P <sub>coll</sub>	8.9	5.25	5.09	-	6.93
14	0.1 loc	6.2	6.1	> P <sub>coll</sub>	8.8	5.08	5.01	-	6.86
14	0.1 gl	6.3	6.0	> P <sub>coll</sub>	8.9	5.17	4.92	-	6.93
Global imperfection, model CYL 1									
2	0.08	> P <sub>coll</sub>	> P <sub>coll</sub>	> P <sub>coll</sub>	3.9	-	-	-	3.04
3	0.42	5.8	5.4	5.7	6.3	4.76	4.43	4.67	4.91
4	0.042	6.9	6.4	> P <sub>coll</sub>	8.9	5.66	5.25	-	6.93
3	0.86	5.0	4.7	4.3	5.2	4.10	3.86	3.52	4.05
Global imperfection, model CYL 2									
2	0.10	> P <sub>coll</sub>	> P <sub>coll</sub>	> P <sub>coll</sub>	3.9	-	-	-	3.04
3	0.68	5.3	4.9	4.8	5.6	4.35	4.02	3.93	4.37
4	0.060	6.8	6.3	> P <sub>coll</sub>	8.9	5.58	5.17	-	6.93
3	1.15	4.6	4.3	3.8	4.7	3.78	3.53	3.12	3.66
Global imperfection, model CYL 5									
2	0.20	> P <sub>coll</sub>	> P <sub>coll</sub>	> P <sub>coll</sub>	3.7	-	-	-	2.89
3	0.60	5.5	5.1	5.0	5.8	4.51	4.19	4.10	4.52
4	0.065	6.8	6.3	> P <sub>coll</sub>	8.9	5.58	5.17	-	6.93
3	1.02	4.8	4.4	4.0	4.9	3.94	3.61	3.28	3.82
Factors unequal to 1.0 are: For axisymmetrical (perfect case): $\beta_{mat} = 0.96$ and $\gamma_{num} = 0.95$ For imperfect cases: $\beta_{mat} = 0.96$ , $\beta_{res-coll} = 0.95$ , $\beta_{int-crit}$ and $\beta_{int-coll} = 0.9$ and $\gamma_{num} = 0.95$									

Table A5.4 Critical and collapse pressures in MPa as calculated by MARC for several imperfections



APPENDIX 6 RESULTS OF STAGE 4, NUMERICAL CALCULATIONS, FINITE CYLINDER

Position	P <sub>calc</sub> BOSOR5 (crit)	P <sub>real</sub> BOSOR5	P <sub>calc</sub> MARC (crit)	P <sub>real</sub> MARC
Midway ring 6/7	7.1	6.48	7.0	6.38
Midway ring 7/8	7.1	6.48	7.0	6.38
Midway ring 8/9	7.1	6.48	7.0	6.38
Midway ring 9/10	7.1	6.48	7.0	6.38
Midw. ring 10/11	6.8	6.20	6.7	6.11
Midw. ring 11/12	7.0	6.38	7.0	6.38
Hull, at ring 6	6.2	5.65	6.4	5.84
Hull, at ring 7	6.2	5.65	6.4	5.84
Hull, at ring 8	6.2	5.65	6.4	5.84
Hull, at ring 9	6.2	5.65	6.4	5.84
Hull, at ring 10	6.1	5.56	6.4	5.84
Hull, at ring 11	7.6	6.93	8.1	7.39
Web ring 6 1)	11.9	10.85	-	-
Web ring 7 1)	11.9	10.85	-	-
Web ring 8 1)	11.9	10.85	-	-
Web ring 9 1)	11.9	10.85	-	-
Web ring 10 1)	11.8	10.76	-	-
Web ring 11 1)	11.3	10.31	-	-
1) is analyses with hull material elastic. End caps start to yield at 30 MPa				
P <sub>collapse</sub>	8.85	8.07	9.4	8.57
P <sub>collapse</sub> 1)	> 55.0	-	-	-
Factors unequal to 1.0: $\beta_{mat} = 0.96$ and $\gamma_{num} = 0.95$				

Table A6.1 Results as calculated with MARC and BOSOR5 (in MPa) for the axisymmetrical state of the finite testcylinders

Elastic buckling						
n	P <sub>calc</sub> BOSOR5	P <sub>real</sub> BOSOR5	P <sub>start</sub> MARC	P <sub>calc</sub> MARC	P <sub>k</sub> live load	P <sub>real</sub> MARC
2	16.7	13.86	15.0	21.696	16.27	13.50
3	11.5	9.54	10.0	13.075	11.62	9.64
4	16.4	13.60	15.0	17.907	16.79	13.93
4	-	-	15.0 1)	23.788	22.30	18.50
5	22.4	18.59	-	-	-	-
6	27.3	22.65	10.0 2)	28.82	28.02	23.25
1): n=4 mode with k=1. 2): calculated with MARC model for n=3. Factors unequal to 1.0: $\beta_{mat} = 0.96$ , $\beta_{shape} = 0.98$ , $\beta_{res} = 0.98$ , $\gamma_{num} = 0.9$						
Plastic buckling (only with BOSOR5)						
2	9.06 3)	7.44				
3	8.73	7.17				
4	9.03 3)	7.41				
3): calculated from eigenvalue at P=8.84 MPa, otherwise $P_k > P_{collapse}$ Factors unequal to 1.0: $\beta_{mat} = 0.96$ , $\beta_{res} = 0.95$ , $\gamma_{num} = 0.9$						

Table A6.2 Elastic and plastic buckling pressures as calculated with MARC and BOSOR5 (in MPa) for the finite testcylinders

Results CYL 1		P <sub>calc</sub> values BOSPOST			P <sub>real</sub> values BOSPOST		
		Critical pressures			Critical pressures		
n	W0	Hull Midbay	Hull At web	Web	Hull Midbay	Hull At web	Web
2	0.08	6.72	6.03	11.11	5.52	4.95	9.12
3	0.42	5.76	5.12	6.21	4.73	4.20	5.10
4	0.042	6.73	6.01	10.99	5.52	4.93	9.02
5	0.025	6.72	6.04	11.11	5.52	4.96	9.12
3	0.86	5.01	4.33	4.72	4.11	3.55	3.87
Results CYL 2		P <sub>calc</sub> values BOSPOST			P <sub>real</sub> values BOSPOST		
		Critical pressures			Critical pressures		
n	W0	Hull Midbay	Hull At web	Web	Hull Midbay	Hull At web	Web
2	0.10	6.71	6.01	11.10	5.51	4.93	9.11
3	0.68	5.28	4.62	5.21	4.33	3.79	4.28
4	0.060	6.73	5.96	10.59	5.52	4.89	8.69
5	0.033	6.73	6.02	11.12	5.52	4.94	9.13
3	1.15	4.64	3.93	4.12	3.81	3.23	3.38
Results CYL 5		P <sub>calc</sub> values BOSPOST			P <sub>real</sub> values BOSPOST		
		Critical pressures			Critical pressures		
n	W0	Hull Midbay	Hull At web	Web	Hull Midbay	Hull At web	Web
2	0.20	6.69	5.92	10.97	5.49	4.86	9.00
3	0.60	5.41	4.76	5.47	4.44	3.91	4.49
4	0.065	6.73	5.95	10.47	5.52	4.88	8.59
5	0.025	6.73	6.04	11.12	5.52	4.96	9.13
3	1.02	4.79	4.10	4.36	3.93	3.37	3.58
Factors unequal to 1.0: $R_{mat} = 0.96$ , $R_{int} = 0.9$ and $\gamma_{num} = 0.95$							

Table A6.3 Results as calculated with BOSPOST in case of a global imperfection, finite cylinders (pressures in MPa, W0 in mm)

Results CYL 1, critical pressures, imperfection n=3, W0 = 0.42 mm								
Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>
Midw web 6/7	6.0	4.92	Hull at web 6	5.6	4.60	Web 6	6.6	5.42
Midw web 7/8	6.2	5.09	Hull at web 7	5.6	4.60	Web 7	6.8	5.58
Midw web 8/9	6.4	5.25	Hull at web 8	5.8	4.76	Web 8	7.0	5.75
Midw web 9/10	6.6	5.42	Hull at web 9	6.0	4.92	Web 9	7.3	5.99
Midw web 10/11	6.6	5.42	Hull at web 10	6.2	5.09	Web 10	*	
Midw 11 - cap	7.0	5.75	Hull at web 11	*		Web 11	*	
Results CYL 1, imperfection n=3, W0 = 0.42 mm, collapse pressure:							7.5	5.85
Results CYL 2, critical pressures, imperfection n=3, W0 = 0.68 mm								
Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>
Midw web 6/7	5.6	4.60	Hull at web 6	5.3	4.35	Web 6	5.6	4.60
Midw web 7/8	5.8	4.76	Hull at web 7	5.3	4.35	Web 7	5.8	4.76
Midw web 8/9	6.0	4.92	Hull at web 8	5.5	4.51	Web 8	6.1	5.00
Midw web 9/10	6.4	5.25	Hull at web 9	5.9	4.84	Web 9	6.6	5.42
Midw web 10/11	6.6	5.42	Hull at web 10	6.3	5.17	Web 10	*	
Midw 11 - cap	6.7	5.50	Hull at web 11	*		Web 11	*	
Results CYL 2, imperfection n=3, W0 = 0.68 mm, collapse pressure:							6.9	5.38
Results CYL 5, critical pressures, imperfection n=2, W0 = 0.20 mm								
Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>
Midw web 6/7	6.8	5.58	Hull at web 6	6.3	5.17	Web 6	*	
Midw web 7/8	6.8	5.58	Hull at web 7	6.3	5.17	Web 7	*	
Midw web 8/9	6.9	5.66	Hull at web 8	6.3	5.17	Web 8	*	
Midw web 9/10	7.0	5.75	Hull at web 9	6.4	5.25	Web 9	*	
Midw web 10/11	6.7	5.50	Hull at web 10	6.4	5.25	Web 10	*	
Midw 11 - cap	6.9	5.66	Hull at web 11	8.0	6.57	Web 11	*	
Results CYL 5, imperfection n=2, W0 = 0.20 mm, collapse pressure:							8.8	6.86



Results CYL 2, critical pressures, imperfection n=3, max. W0 = 1.15 mm								
Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>
Midw web 6/7	5.0	4.10	Hull at web 6	4.8	3.94	Web 6	4.4	3.61
Midw web 7/8	5.2	4.27	Hull at web 7	4.8	3.94	Web 7	4.5	3.69
Midw web 8/9	5.6	4.60	Hull at web 8	5.0	4.10	Web 8	4.9	4.02
Midw web 9/10	6.0	4.92	Hull at web 9	5.5	4.51	Web 9	5.6	4.60
Midw web 10/11	*		Hull at web 10	*		Web 10	6.0	4.92
Midw 11 - cap	*		Hull at web 11	*		Web 11	*	
Results CYL 2, imperfection n=3, max. W0 = 1.15 mm, collapse pressure:							6.1	4.76
Results CYL 5, critical pressures, imperfection n=3, max. W0 = 1.02 mm								
Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>	Position	P <sub>calc</sub>	P <sub>real</sub>
Midw web 6/7	5.1	4.19	Hull at web 6	4.8	3.94	Web 6	4.7	3.86
Midw web 7/8	5.3	4.35	Hull at web 7	4.8	3.94	Web 7	4.8	3.94
Midw web 8/9	5.7	4.68	Hull at web 8	5.1	4.19	Web 8	5.1	4.19
Midw web 9/10	6.2	5.09	Hull at web 9	5.7	4.68	Web 9	5.7	4.68
Midw web 10/11	*		Hull at web 10	6.2	5.09	Web 10	6.2	5.09
Midw 11 - cap	*		Hull at web 11	*		Web 11	*	
Results CYL 5, imperfection n=3, max. W0 = 1.02 mm, collapse pressure:							6.3	4.91

Table A6.4 Results as calculated with MARC (in MPa) in case of a global imperfection (\*: larger than P<sub>collapse</sub>. Factors unequal to 1.0:  $\beta_{mat} = 0.96$ ,  $\beta_{res-coll} = 0.95$ ,  $\beta_{int} = 0.9$  and  $\gamma_{num} = 0.95$ )

#### APPENDIX 7 COMPARISON MARCK42, MARCK5, DIANA 5.0

During this project, some analyses have been carried out using DIANA 5.0, to compare results with MARC. An important disadvantage is the relatively bad data management system of DIANA 5.0, resulting in the use of considerably larger amounts of disk spaces. A comparison of the used disk space (increase from MARCK42 to MARCK5 caused by change from partly double precision to complete double precision):

MARCK42, 9000 nodes, 11 layers: 890 Mb.

MARCK5, 9000 nodes, 11 layers: 605 Mb.

DIANA 5.0, 2000 nodes, 7 layers: 700 Mb, but necessary to decrease data storage after each increment.

Considering that disk space generally has a more or less quadratic dependence of the number of degrees of freedom, DIANA 5.0 is supposed to use about 35 Gb for the 9000 nodes, 11 layers model  $((9000/2000 * 11/7)**2 * 700 \text{ Mb} = 35 \text{ Gb})$ .

Besides, output results from DIANA 5.0 use considerably more disk space. For example, stresses at nodes:

MARC:

- possible to ask for only one component.
- stresses at nodes already averaged.
- about 6 characters for stress component, and 4 characters for the node number.

DIANA:

- not possible to ask for only one component.
- stresses not averaged. So for a node present in four elements stresses are given four times.
- each time AXES (directions) are printed (header plus nine coordinates (for three directions) in E16 format).
- stresses are given in E16 format, each line is filled up to 80 characters. Each time the name of the stress component is printed.

Generally results as calculated with DIANA 5.0 shows a good correspondence with MARC and BOSOR5 results. Some other remarks with respect to DIANA 5.0 are:

- It is possible to perform an eigenvalue analysis starting from a nonlinear solution at the equilibrium path, but these results are completely wrong. So the analysis as performed for the investigation of interaction (eigenvalue analysis for imperfect cylinder) can not be carried out with DIANA 5.0.
- The used DIANA element (Q20SH, linear curved shell element) has a worse performance than MARC element 75. More elements are necessary to obtain a solution which shows no further convergence with an increasing number of elements. This does not hold if DIANA element CQ40S (quadratic curved shell element) is used, but of course this still results in more degrees of freedom.
- The follower force effect is only available for the quadratic shell element (CQ40S).
- Follower force effect for the pressure load at the hull in combination with the axial forces PR/2 is not possible. Modelling PR/2 as nodal forces, these forces are removed from the load vector, and modelling PR/2 as element edge loads results in an error.

It is noted that these remarks will be discussed with the development department of the DIANA finite element package.